

# Relaxed Bayesian phylogenetics

## Molecular Clocks and Calibration

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Workshop on Molecular Evolution  
Cesky Krumlov, 30th Jan 2015

- 1 BEAST
- 2 Tree Space
- 3 Bayesian phylogenetics
- 4 Clocks and calibrations
- 5 Relaxed phylogenetics
- 6 Total evidence and fossilized birth-death tree prior

**BEAST**

# BEAST

BEAST focuses on **time-trees** (phylochronologies); both species trees and gene trees

Currently useful for

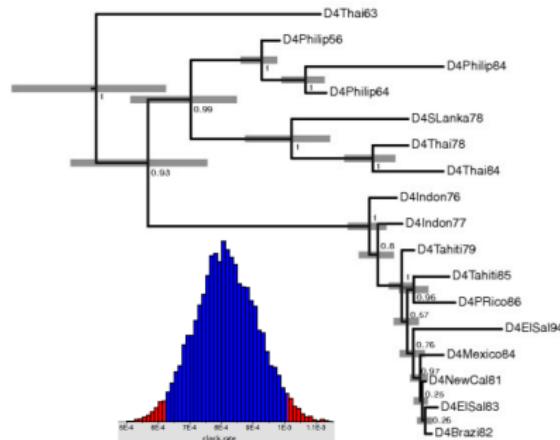
- Divergence time dating
- Estimating phylogenies under relaxed clock models
- Single population coalescent reconstruction
- Estimation of rates from viruses or ancient DNA
- Co-estimation of species trees and gene trees
- Automatic partitioning and substitution model selection

Working on

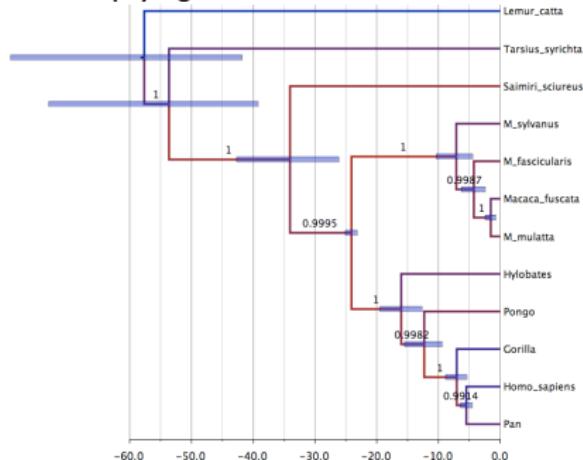
- More tree priors, more clock models, more substitution models
- More efficient tree sampling techniques (HMC)
- Phylodynamical models, host-pathogen co-phylogeny models

# BEAST 1.4.8

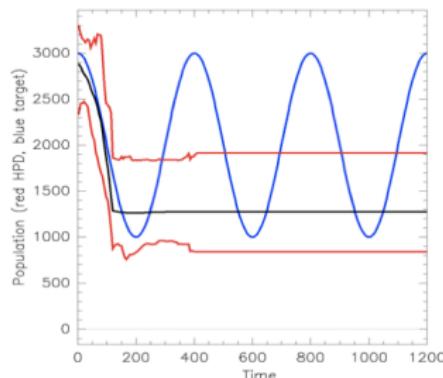
## Rates/dates from serially sampled data



## Relaxed phylogenetics

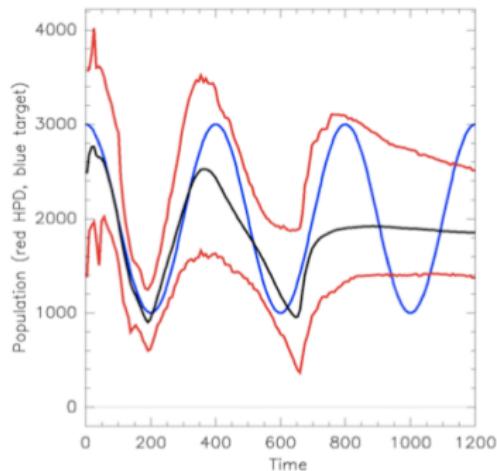


## Estimating population size and changes



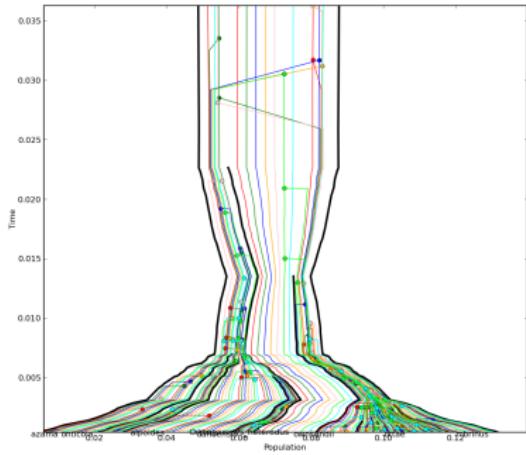
# BEAST 1.6

Bayesian skyline plots and coalescent models with multiple loci



Bayesian skyride

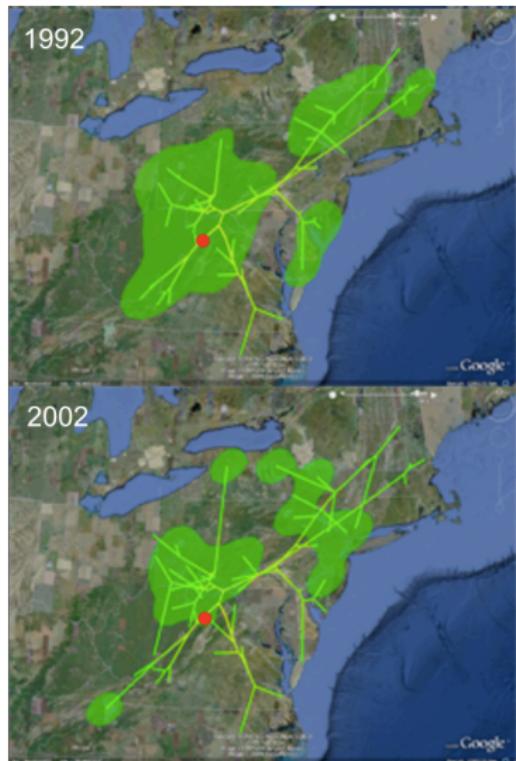
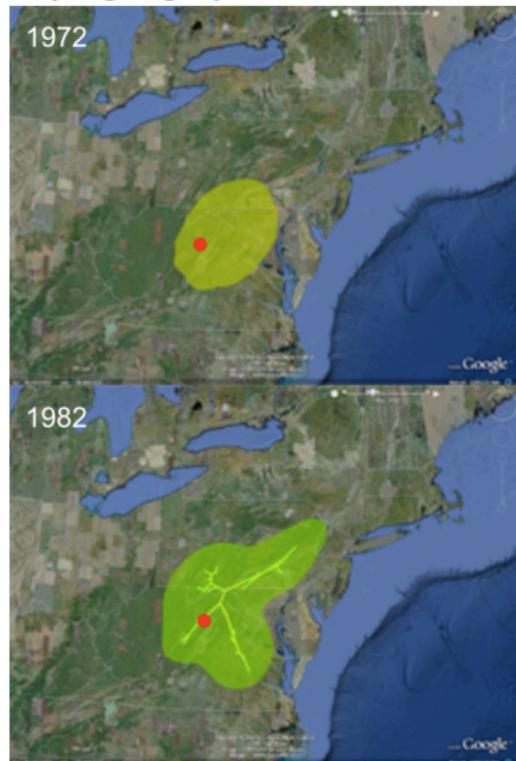
Coestimation of species tree and gene trees



Generalized partitioning

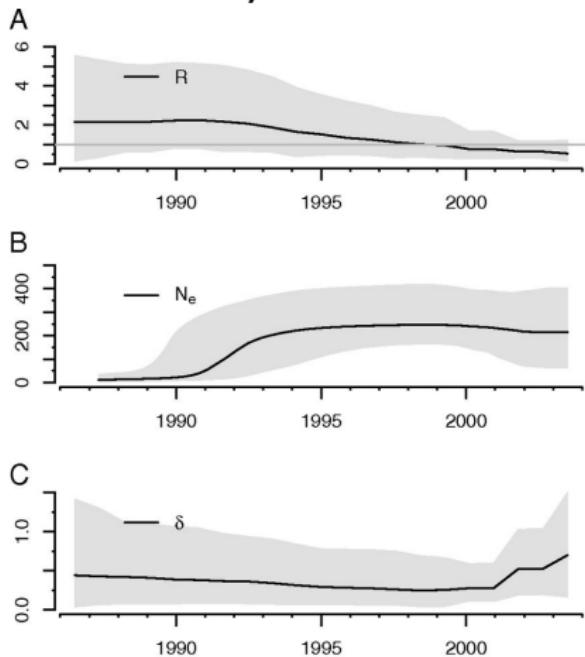
# BEAST 1.8

## Phylogeographic models

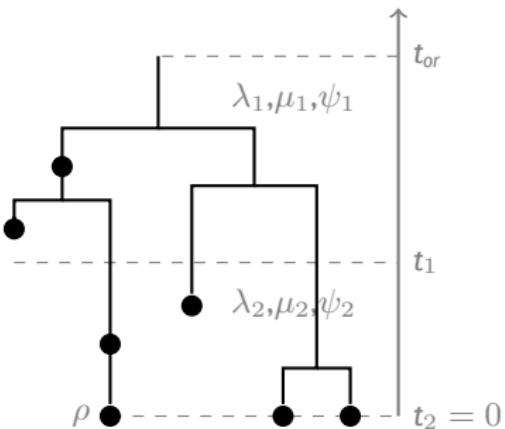


# BEAST 2.2

## Birth-death-skyline models



## Fossilized birth-death models

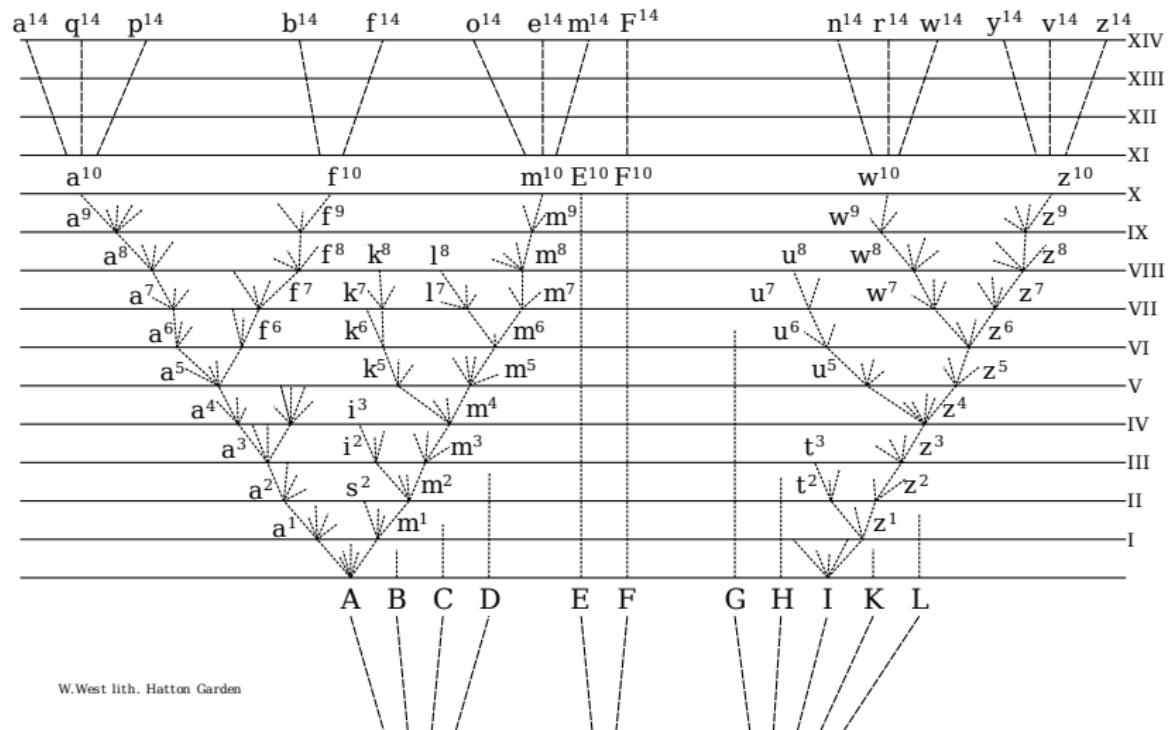


and lots of others (e.g. *Dirichlet process site partition model averaging*)

# Tree Space

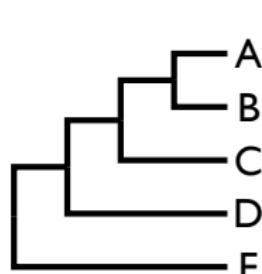
# Darwin's Tree of Life

The only illustration in the *Origin of Species* (Darwin, 1859)

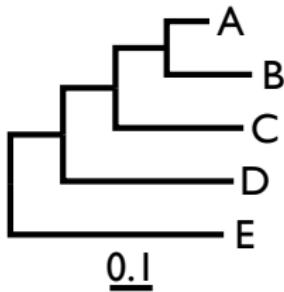


# Types of phylogenies and representations

rooted trees

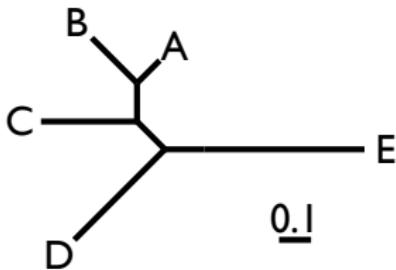


(a) cladogram



(b) phylogram

unrooted tree



(c) unrooted tree

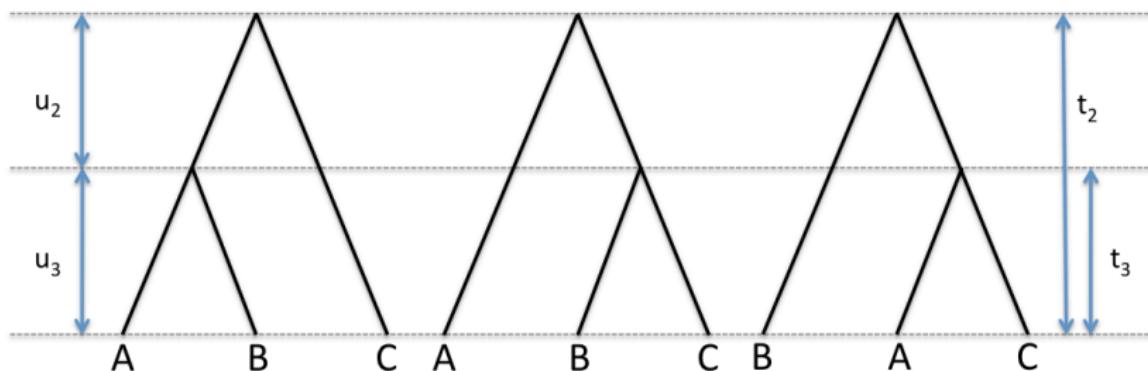
$((((A, B), C), D), E);$

$((((A:0.1, B:0.2):0.12, C:0.3):0.123, D:0.4):0.1234, E:0.5);$

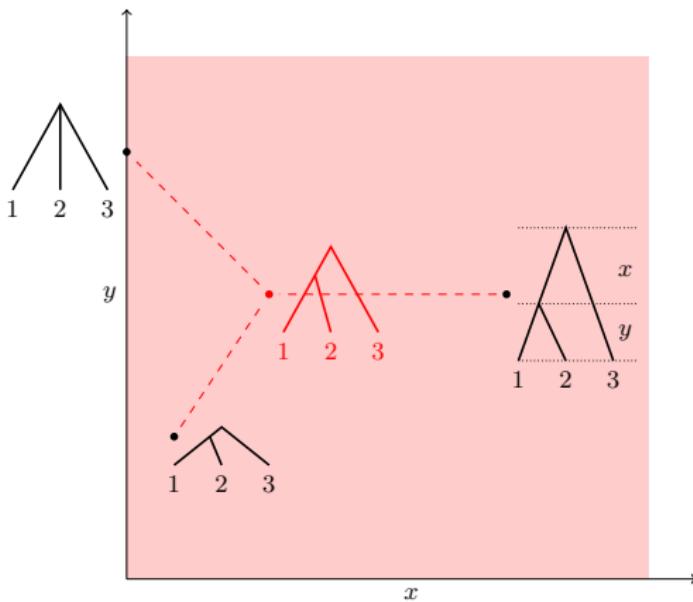
branches (edges) and their lengths, nodes, tips (leaves)

## The tip-labeled time-tree

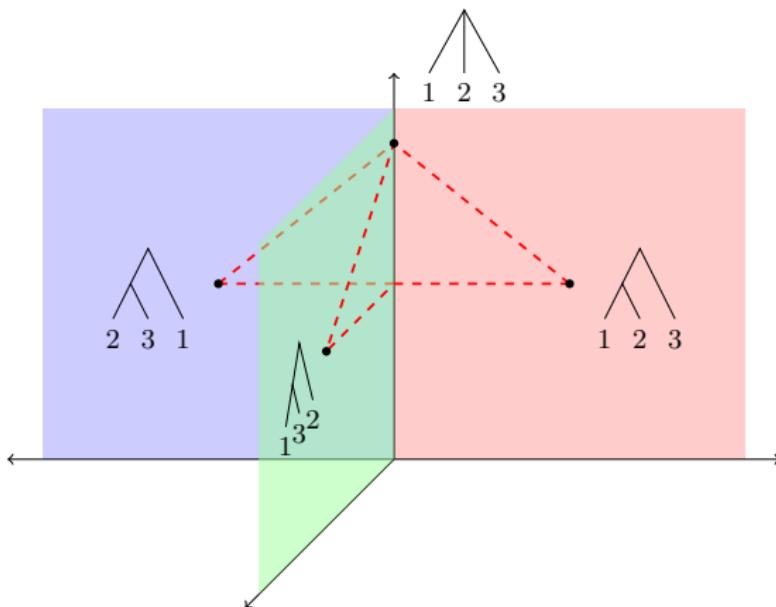
A tip-labeled time-tree is described by a *tip-labeled ranked topology* of size  $k$  and *coalescent times*,  $\mathbf{u} = \{u_2, \dots, u_k\}$ .



These time-trees of size 3 can be interpreted as describing the possible alternative evolutionary histories for three species or (uniparental) ancestries of the three individuals represented by the labeled tips.



**Figure:** A Euclidean two-dimensional space representing the space of all possible time-trees for the topology  $((1,2),3)$ . There are two parameters,  $x$  and  $y$ , one for each of the two inter-coalescent intervals, the sum of which is the age of the root ( $t_{\text{root}} = x + y$ ). Three trees are displayed, along with their arithmetic mean tree, also called the *centroid*. The dashed lines show the path connecting each of the three trees to the mean tree by the shortest distance (i.e. their deviations from the mean).



**Figure:**  $\tau_3$ , the simplest non-trivial tree space (for time-trees), representing the space of time-trees for  $n = 3$  taxa sampled contemporaneously. Each of the three non-degenerate tree topologies is represented by a two-dimensional Euclidean space (as illustrated in Figure 1) and these subspaces meet at a single shared edge representing the star tree, which is a one-dimensional subspace and thus has a single parameter (the age of the root). The dashed lines shows the paths of shortest distance between the four displayed trees.

## Another space of tip-labeled time-trees of size 3

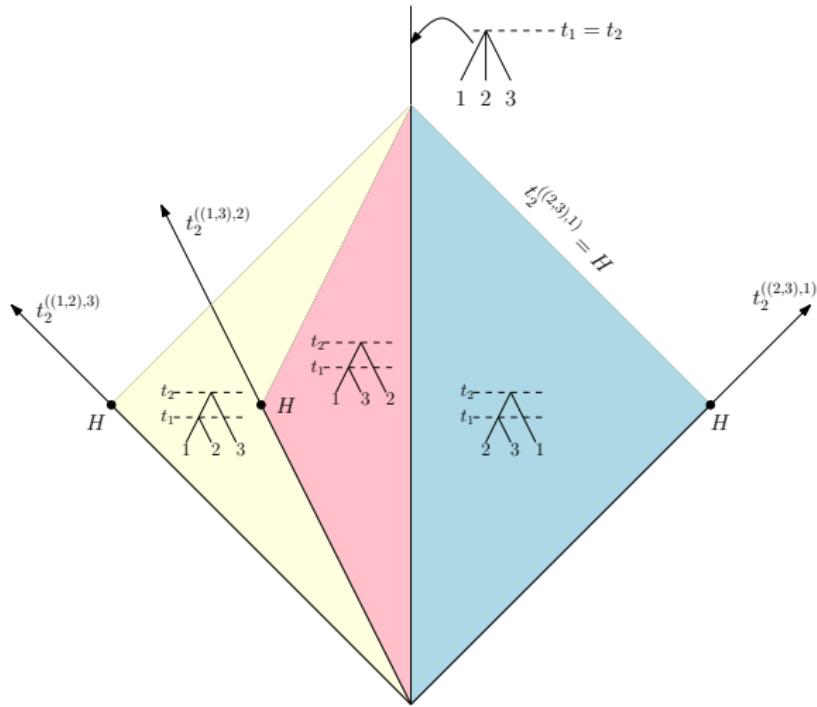


Figure: Space  $\mathbb{T}_3$ .

# A space of tip-labeled time-trees of size 4

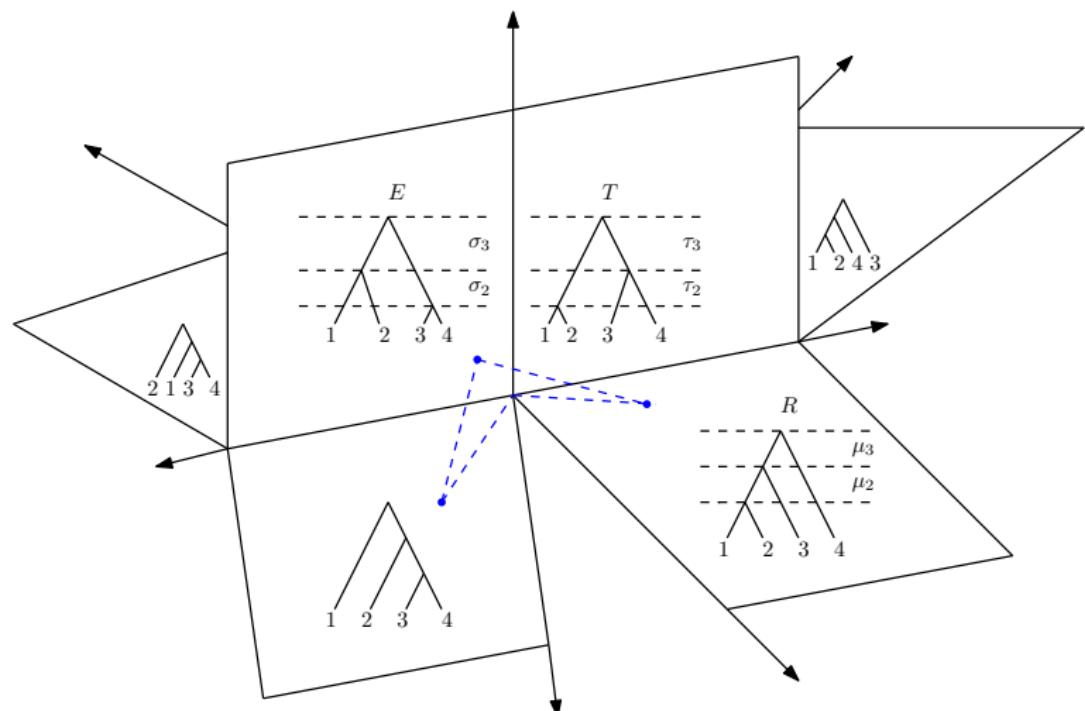


Figure: Three-dimensional projection of 4-dimensional  $\tau$ -space 4.

## Unranked tree topologies of size 4



# How many trees are there?

For  $n$  species there are

$$T_n = 1 \times 3 \times 5 \times \cdots \times (2n - 3) = \frac{(2n-3)!}{(n-2)!2^{n-2}}$$

rooted, tip-labelled binary trees:

$n$	#trees	
4	15	enumerable by hand
5	105	enumerable by hand on a rainy day
6	945	enumerable by computer
7	10395	still searchable very quickly on computer
8	135135	about the number of hairs on your head
9	2027025	greater than the population of Auckland
10	34459425	$\approx$ upper limit for exhaustive search
20	$8.20 \times 10^{21}$	$\approx$ upper limit of branch-and-bound searching
48	$3.21 \times 10^{70}$	$\approx$ the number of particles in the Universe
136	$2.11 \times 10^{267}$	number of trees to choose from in the “Out of Africa” data (Vigilant et al. 1991)

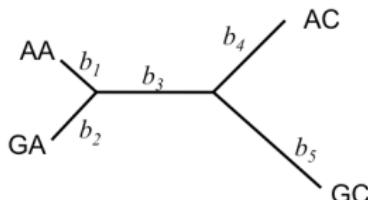
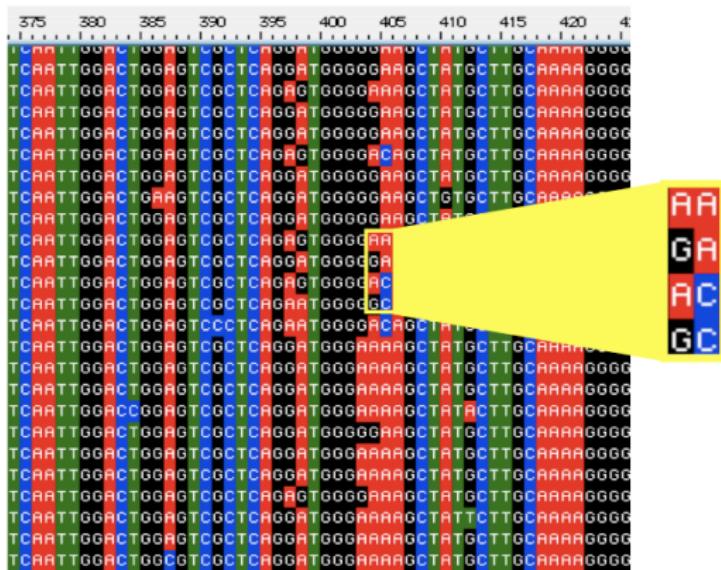
## Counting different types of rooted trees

$n$	#shapes	#trees, $ \mathcal{T}_n $	#ranked trees	#fully ranked trees
2	1	1	1	1
3	1	3	3	4
4	2	15	18	34
5	3	105	180	496
6	6	945	2700	11056
7	11	10395	56700	349504
8	23	135135	1587600	14873104
9	46	2027025	57153600	819786496
10	98	34459425	2571912000	56814228736

**Table:** The number of unlabeled rooted tree shapes, the number of labelled rooted trees, the number of labelled ranked trees (on contemporaneous tips), and the number of fully-ranked trees (on distinctly-timed tips) as a function of the number of taxa,  $n$ .

# Bayesian phylogenetics

# Felsenstein's likelihood (1981)



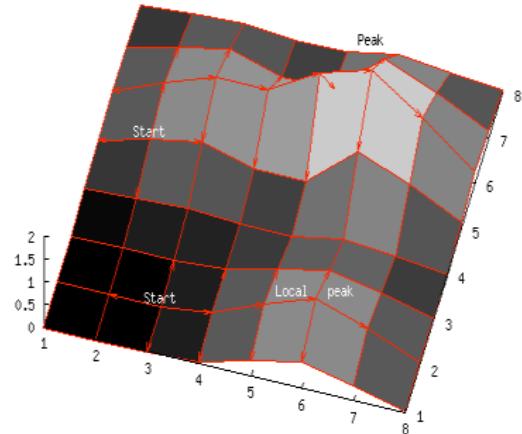
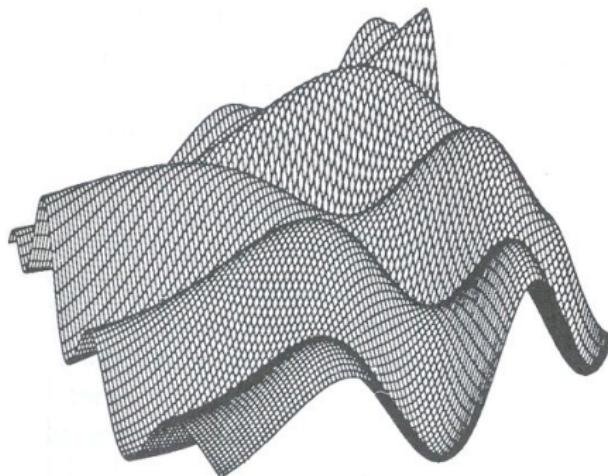
$$L(T) = \Pr\{D|T, Q\}$$

The probability of the data,  $\Pr\{D|T, Q\}$  can be efficiently calculated given a phylogenetic tree ( $T$ ), and a **probabilistic model** of molecular evolution ( $Q$ ).

In statistical phylogenetics, branch lengths are traditionally unconstrained.

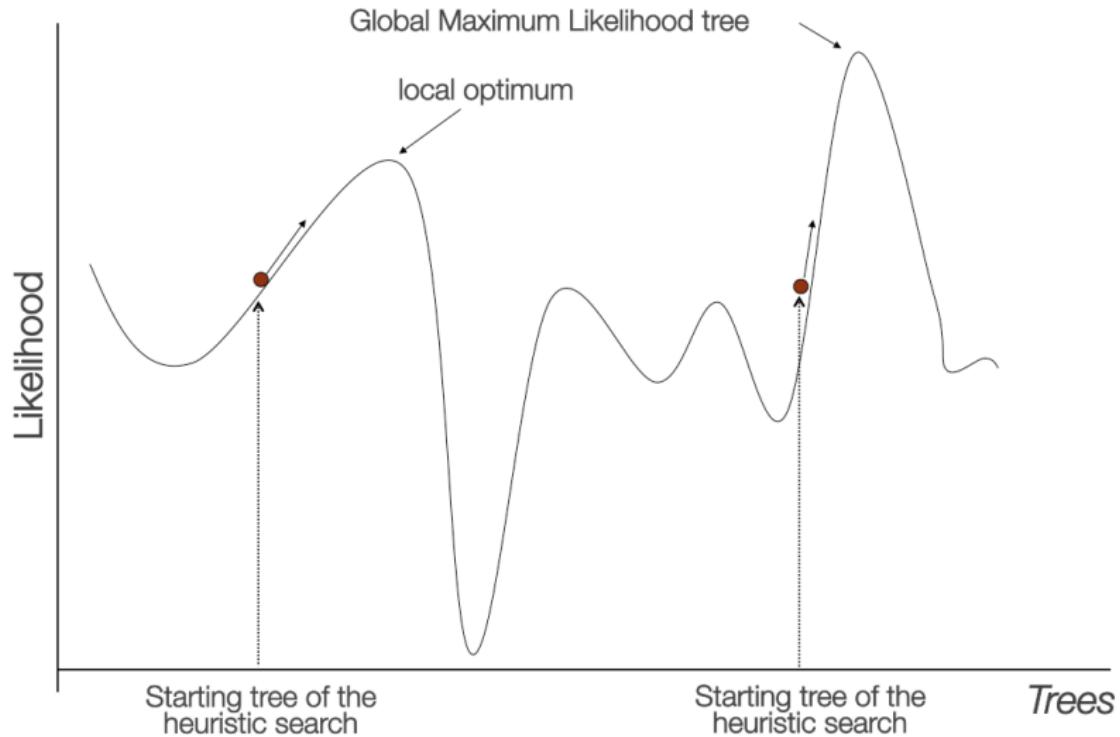
## Tree space as a hilly landscape

The space of all possible trees can be visualized as a hilly landscape. Nearby points in this landscape represent similar trees, and the height of the landscape is the probability of the tree at that point.



- This space can be **sampling** in a Bayesian analysis with MCMC
- The peak can be identified by a **search algorithm** in the context of maximum likelihoods

## Local tree search and multiple optima



## Bayes rule in statistics

$$Pr(\theta|D) = \frac{Pr(D|\theta)Pr(\theta)}{Pr(D)}$$

where

- $P(D|\theta)$  is the **likelihood**,
- $Pr(\theta)$  is the **prior** distribution and
- $Pr(\theta|D)$  is the **posterior** distribution.
- $Pr(D)$  is the **marginal likelihood** of the data.

## Bayes rule in phylogenetics

$$p(T, Q|D) = \frac{Pr\{D|T, Q\}p(T)p(Q)}{Pr\{D\}}$$

where

- $Pr(D|T, Q)$  is Felsenstein's likelihood,
- $p(T)$  is the prior distribution on phylogenetic trees,
- $p(Q)$  is the prior distribution on the model of evolution and
- $p(T, Q|D)$  is the posterior distribution
- $Pr(D)$  is the marginal likelihood of the data.

## Bayesian reconstruction of phylogenetic trees

Yang & Rannala (1997), Mau, Newton & Larget (1998)

In the context of Bayesian phylogenetics, what we want to compute is the **probability of the tree** given the data.

We can compute that from the **likelihood** using **Bayes Theorem**:

$$\text{Posterior probability } P(\Delta \mid 1 \text{ A } 2 \text{ T } 3 \text{ C } 4 \text{ G }) = \frac{\text{Likelihood } Pr(1 \text{ A } 2 \text{ T } 3 \text{ C } 4 \text{ G } \mid \Delta) \text{ Prior Probability } P(\Delta)}{\text{Pr}(1 \text{ A } 2 \text{ T } 3 \text{ C } 4 \text{ G } \mid \text{CATTGACATGATGAAATTAAT})}$$

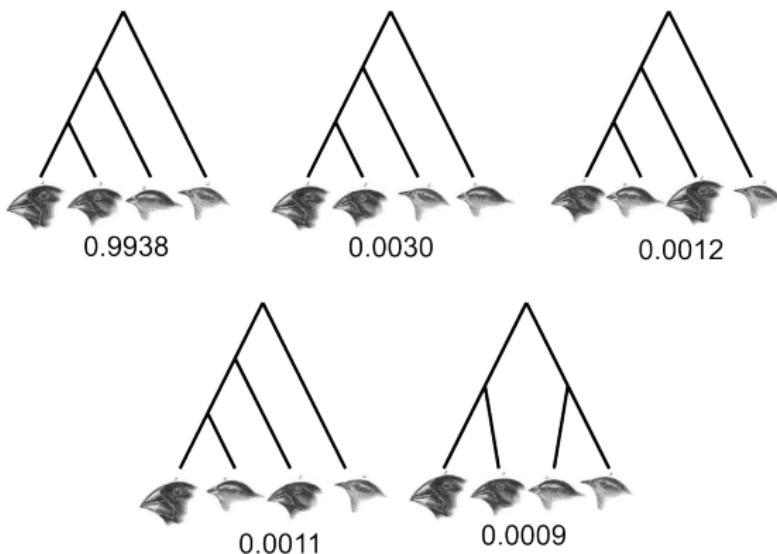
This is known as the **Posterior probability** of the tree. Another method of reconstructing the evolutionary history is then to find the tree that has the **Maximum Posterior probability**.

## Bayesian Phylogenetics

- The output of a Bayesian evolutionary analysis is a **probability distribution on trees and parameter values**.
- For phylogenetics the tree topology is the object of interest. The substitution parameters and tree prior parameters are a nuisance that we average over using MCMC and then ignore.
- For population genetics the tree and substitution parameters are a nuisance that we average over and then ignore, focusing instead on the population parameters.
- Often a more specific hypothesis is of interest (like “Did this adaptive radiation predate the Miocene?”) and then the result of the analysis should be the testing of this hypothesis, averaged over all trees and parameter values, weighted by their probability given the data.

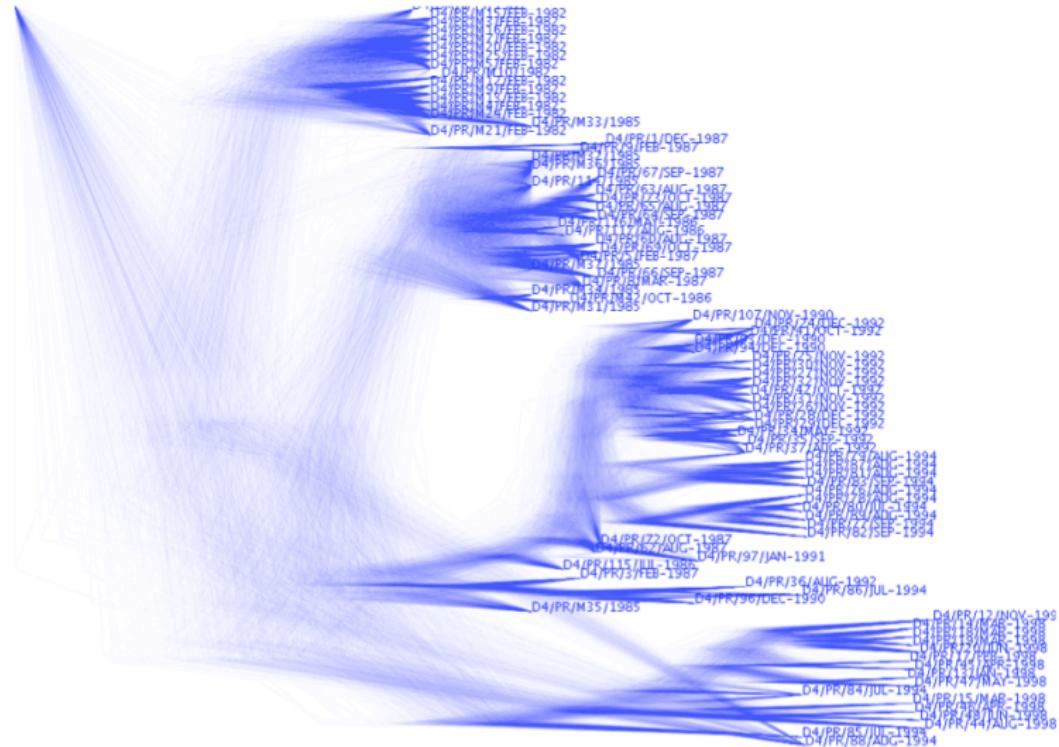
# The Posterior Distribution on Darwin's Finches

1 ATAACCTTCA TTG TAGA TAA TAAT  
2 CTAACCTTCA TTG TAGA TAA TAAT  
3 ACAGCCTCA TTG TGGACCGACAAT  
4 ATGGTCCCT - CCAGAAGCAGTG - C



This posterior probability distribution was computed using an algorithm called **Markov chain Monte Carlo** implemented in the BEAST software package (Drummond & Rambaut, 2007).

## The posterior distribution for larger trees



## Elaborating the model

Basic model: (posterior proportional to likelihood  $\times$  prior)

$$p(T|D) \propto \Pr\{D|T\}p(T)$$

Substitution model estimation:

$$p(T, Q|D) \propto \Pr\{D|T, Q\}p(T)p(Q)$$

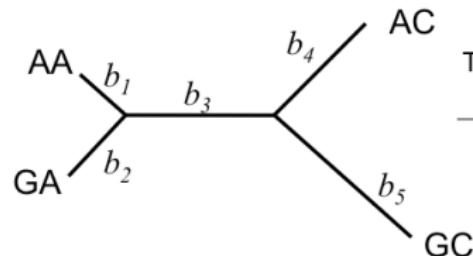
Substitution model and parametric tree prior:

$$p(T, Q, \theta|D) \propto \Pr\{D|T, Q\}p(T|\theta)p(Q)p(\theta)$$

# Clocks and calibrations

# The molecular clock constraint

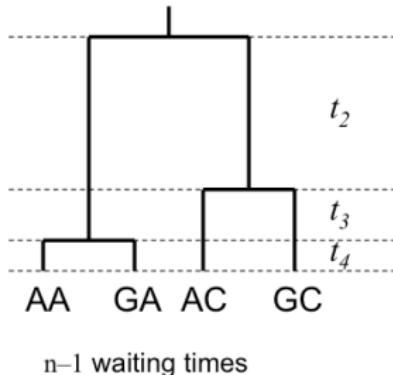
$T$



2n-3 branch lengths

$g$

The “molecular clock”  
constraint



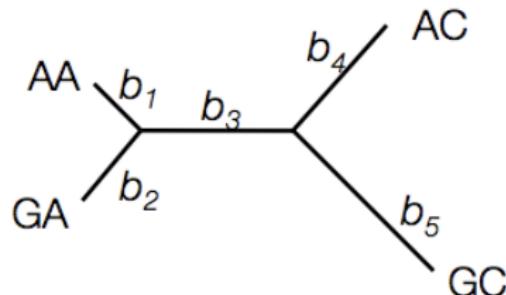
n-1 waiting times

Standard BEAST model:

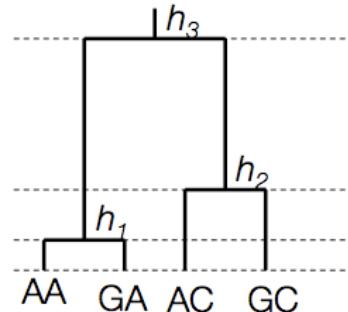
$$p(g, Q, \theta | D) \propto \Pr\{D|g, Q\} p(g|\theta) p(Q|\theta)$$

The joint posterior probability of the **rooted** time-tree ( $g$ ) the substitution matrix ( $Q$ ) and the tree prior parameters ( $\theta$ ) is sampled using Markov chain Monte Carlo (Drummond *et al*, 2002; 2006)

## Model assumptions



- Product of rate and time (branch length) is independent and identically distributed among branches.
- The root of the tree could be anywhere with equal probability.
- Topology implies nothing about individual branch lengths.



- Rate of evolution is the same on all branches.
- The root of the tree is equidistant from all tips.
- Topology constrains branch lengths (e.g. two branches in a cherry must be of equal length)

## Calibration via a global molecular clock

Basic model: (Tree in expected substitutions per site)

$$p(\mathbf{g}, \theta | D) \propto \Pr\{D|\mathbf{g}\} p(\mathbf{g}|\theta) p(\theta)$$

Fix (i.e. condition on) the global rate to  $\mu$ :

$$p(\mathbf{g}, \theta | D) \propto \Pr\{D|\mu \times \mathbf{g}\} p(\mathbf{g}|\theta) p(\theta)$$

Estimate the global rate:

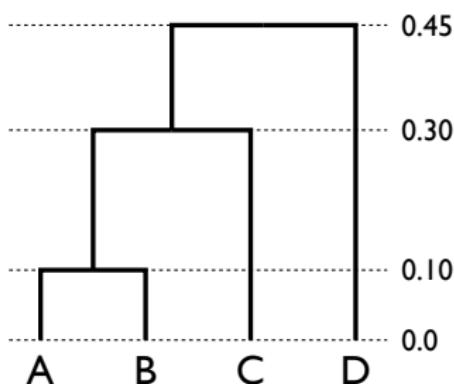
$$p(\mathbf{g}, \mu, \theta | D) \propto \Pr\{D|\mu \times \mathbf{g}\} p(\mathbf{g}|\theta) p(\theta) p(\mu)$$

In the models above the parameters related to the details of the substitution process ( $Q$ ) have been suppressed for simplicity.

# Genetic distance = rate × time

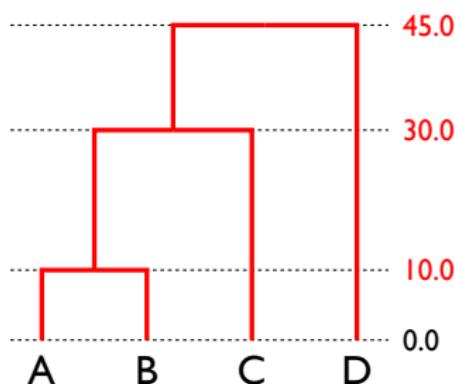
Strict molecular clock

$$T = \mu \times g$$



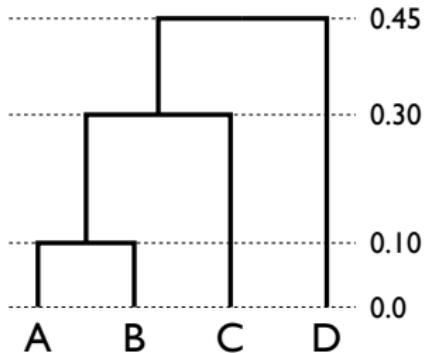
“substitution tree”

evolutionary rate  
substitutions / site / unit time

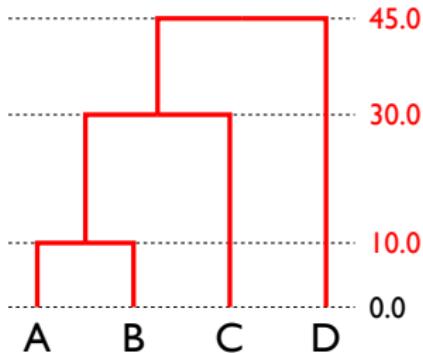


time tree

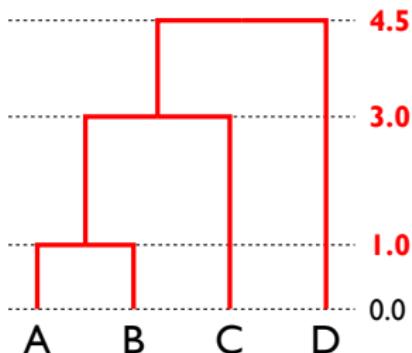
## Non-identifiability of rate and times



= 0.01 ×



= 0.1 ×



“substitution tree”

evolutionary rate  
substitutions / site / unit time

time tree

## A simple calibration is not simple

Consider the simplest type of calibration to admit uncertainty: the placement of an upper and a lower limit on the age of a single calibrated divergence ( $h_C$ ) in the tree:

$$f(h_C) = \begin{cases} 1/(u - l) & l \leq h_C \leq u \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

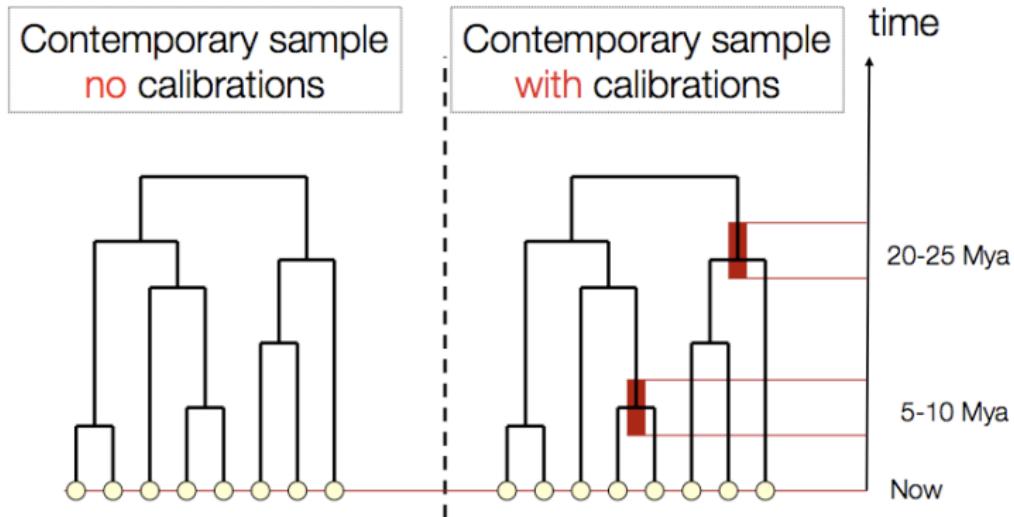
This calibration already has two quite distinct interpretations. One interpretation is that the resulting marginal prior distribution on the calibrated divergence should obey the tree process prior ( $f_G$ , e.g. Yule or Birth-death) but be **constrained** to be within the upper and lower bounds:

$$\rho_G(g|\theta) \propto f_G(g|\theta)f(h_C), \quad (2)$$

Alternatively, the marginal prior of  $h_C$  is uniform and conditioned on:

$$\rho_G(g|\theta) \propto f_G(g_{-h_C}|\theta, h_C)f(h_C), \quad (3)$$

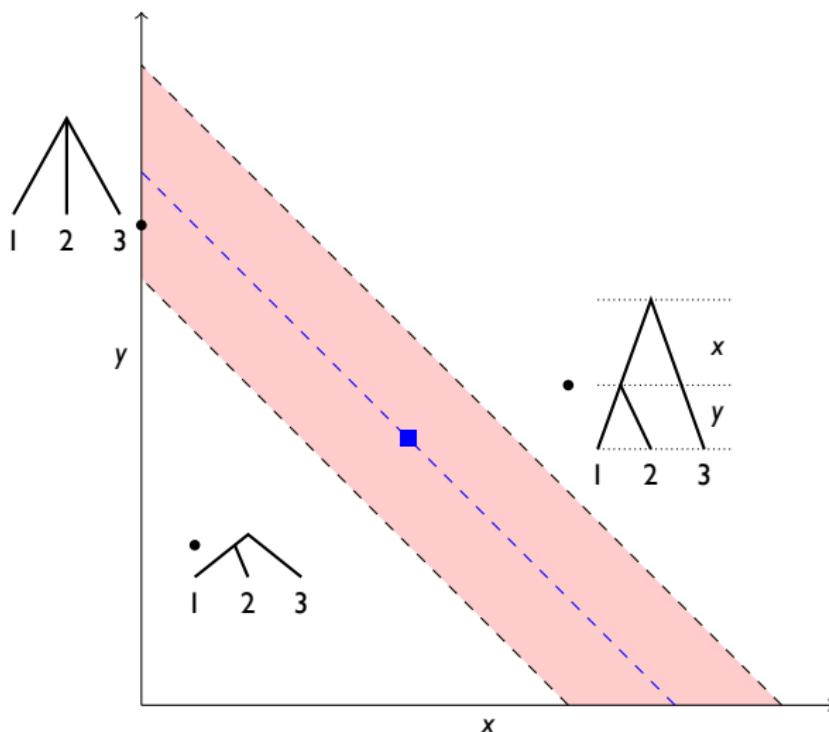
## Absolute time via calibrations



Let  $\rho_G(g|\theta)$  be “calibrated”  $f_G(g|\theta)$  and estimate the rate,  $\mu$ :

$$p(\mu, g, \theta | D) \propto \Pr\{D | \mu \times g\} \rho_G(g|\theta) f_N(\theta) f_M(\mu)$$

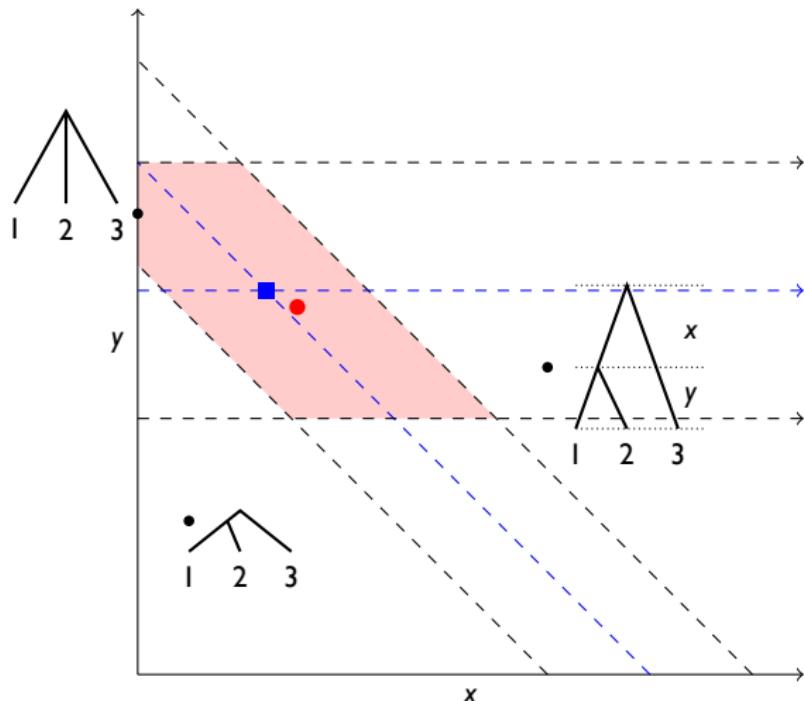
## Calibrating tree space



Single calibration on the root height:  $8 < x + y < 12$

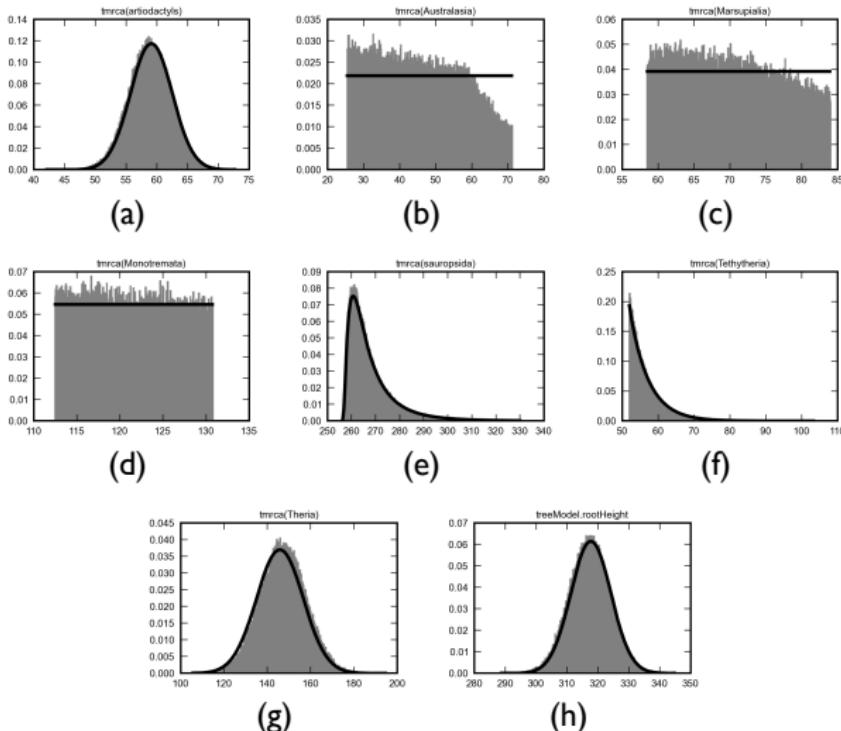
# Calibrating tree space

Two calibrations is even less simple!

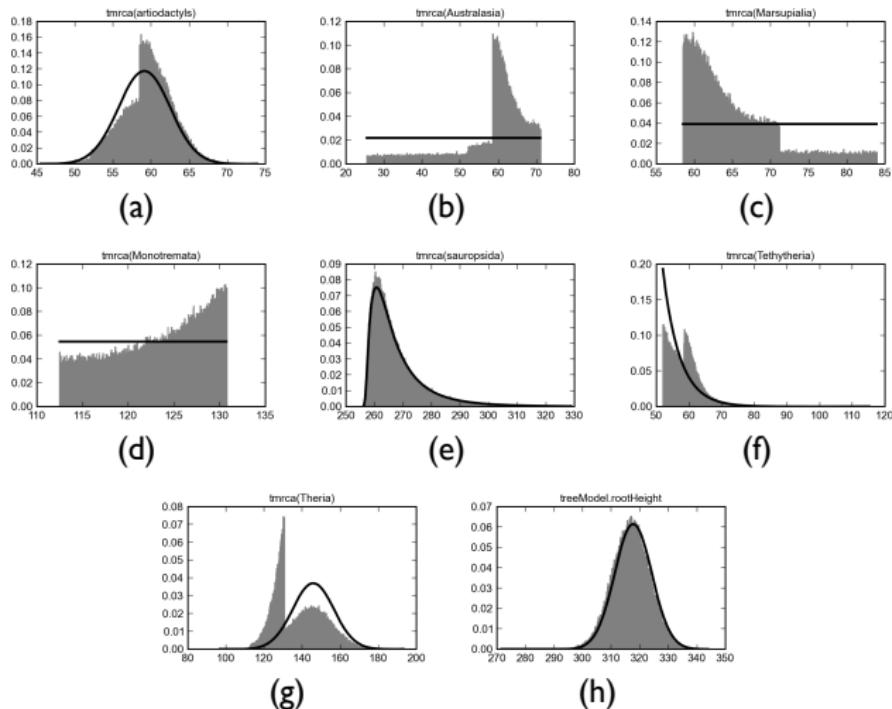


First calibration:  $8 < x + y < 12$

Second calibration:  $5 < y < 10$



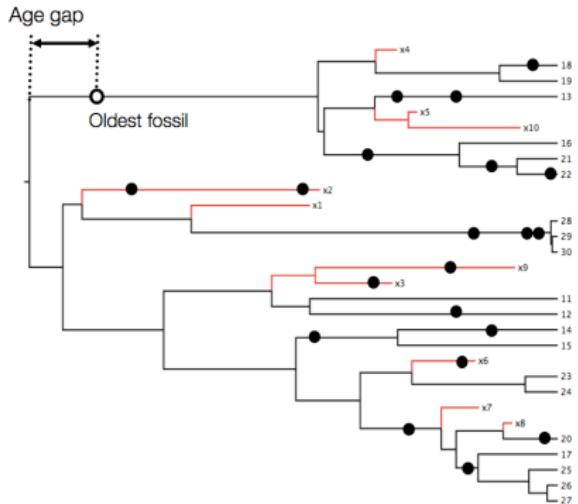
**Figure:** A simple construction of calibrated tree prior:  $\rho_G(g) \propto f_G(g) \times \prod_{i=1}^k f_i(s_i)$ . Where  $f_i()$  is the univariate "calibration density" for the divergence time of the  $i$ 'th calibrated node in the tree. Monophyly is enforced for each calibrated node.



**Figure:** The marginal prior distributions that result from BEAST (gray) versus calibration densities (black) specified for the calibrated nodes from [?]. The marginal prior distributions were obtained from a MCMC run using the prior only.

*How do I pick the calibration density?*

# Modeling the Fossil Age Gap



What is the probability distribution of the age gap?



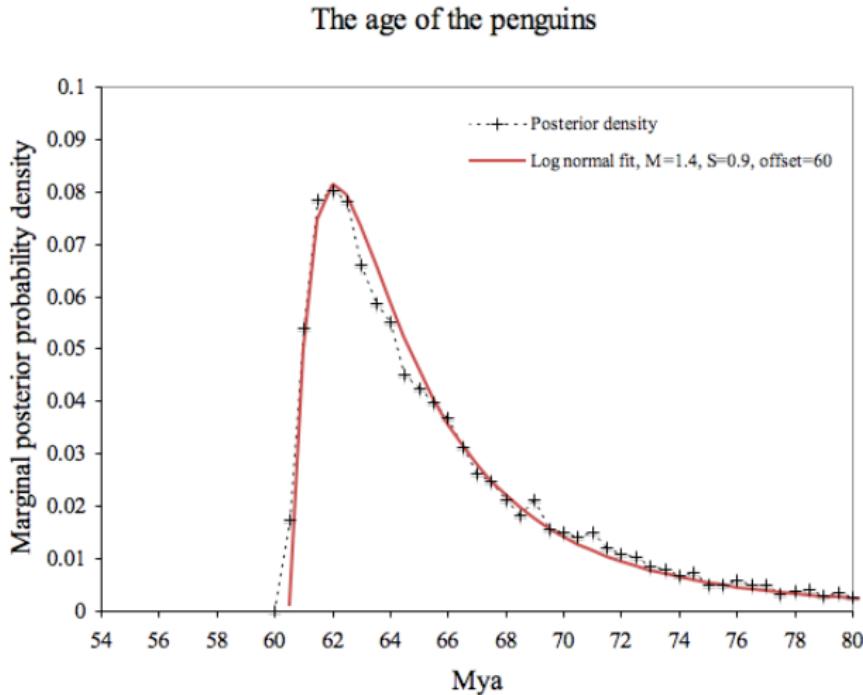
60-61.5 Myr penguin

Prof. Ewan Fordyce with reconstruction of *Waimanu tuatahi*

Current day penguin species: 20

Number of independent penguin fossils with good geological age from all ages: 20-60

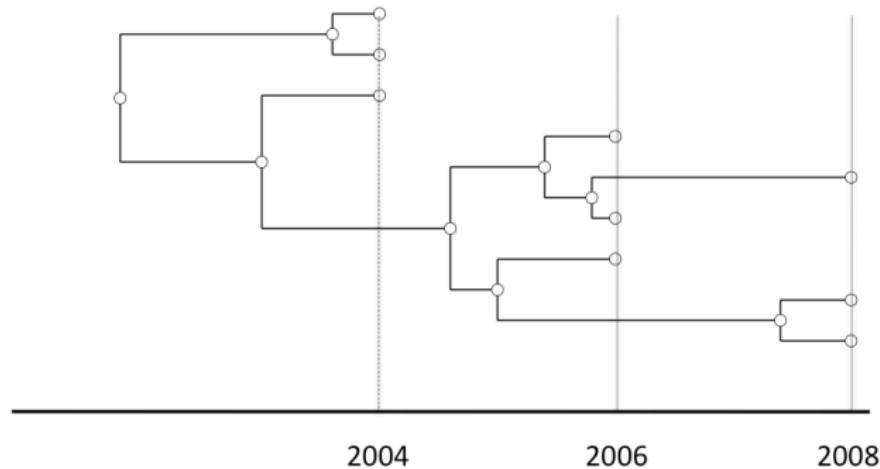
# The posterior estimate of the age of penguins



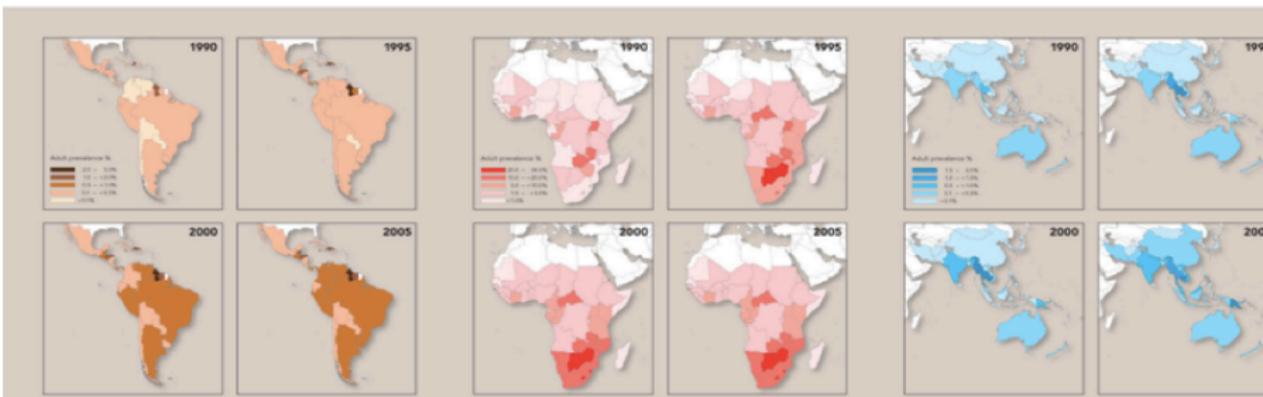
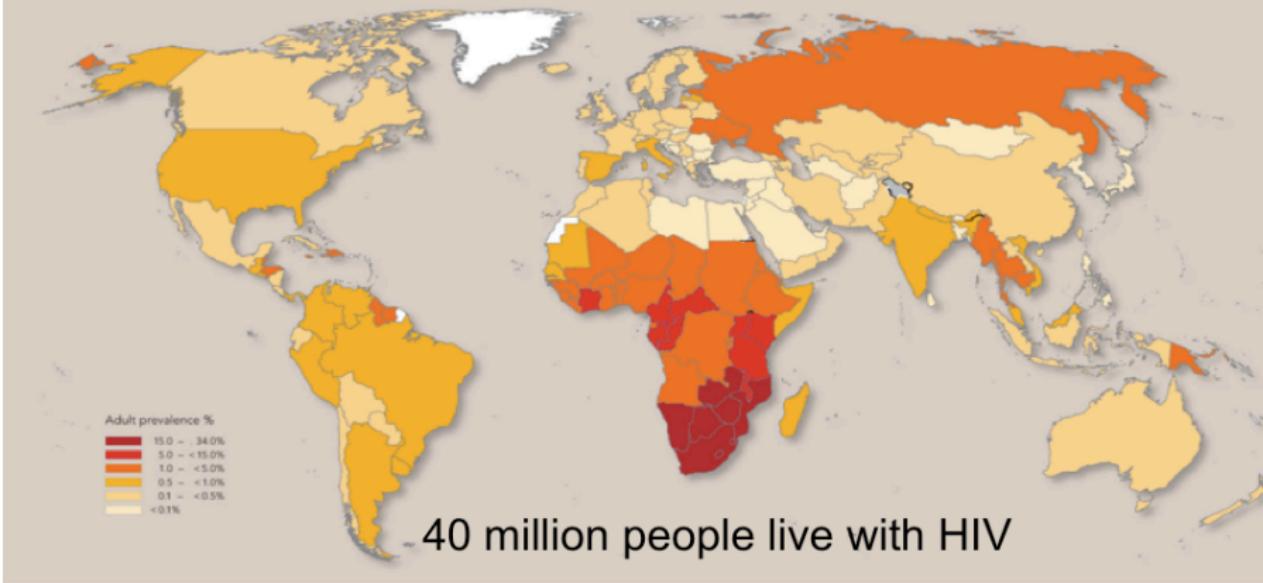
# Evolution is happening right now!

Rodrigo and Felsenstein, 1999; Drummond *et al*, 2002

Many pathogens, such as HIV, Hepatitis C and Influenza A, evolve very rapidly, so that samples of the virus population from different times directly reveal evolutionary change.

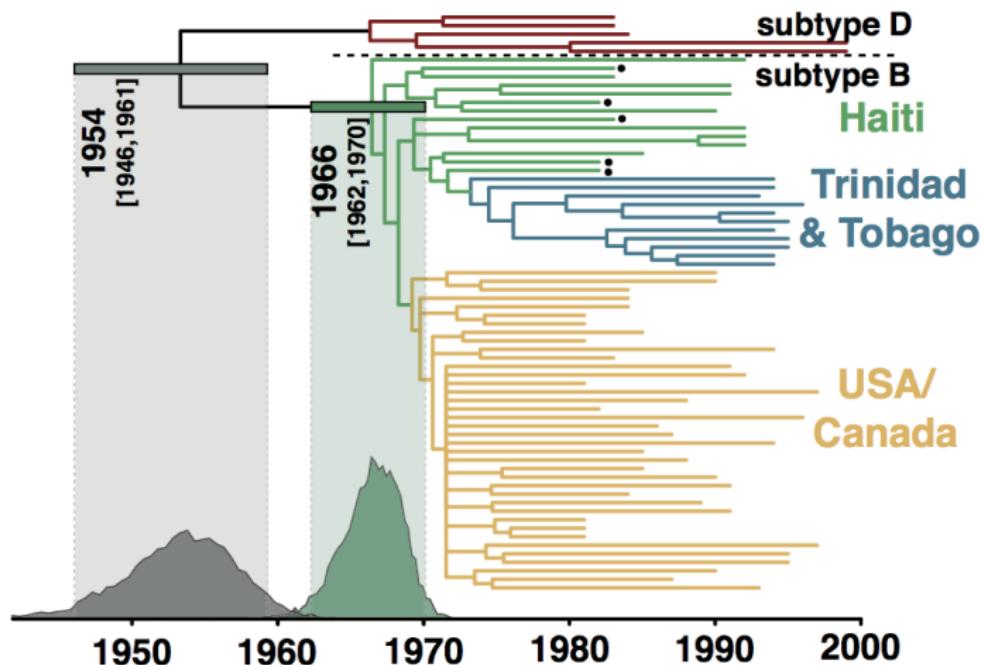


In fact it becomes possible to **calibrate** the tree and thus place the tree on a time scale - by constraining the tips to known sampling times



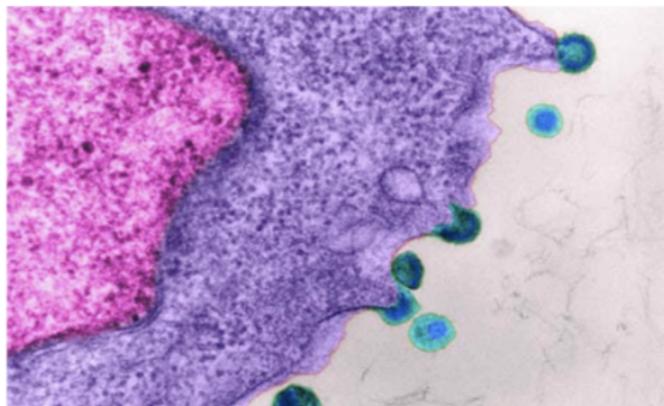
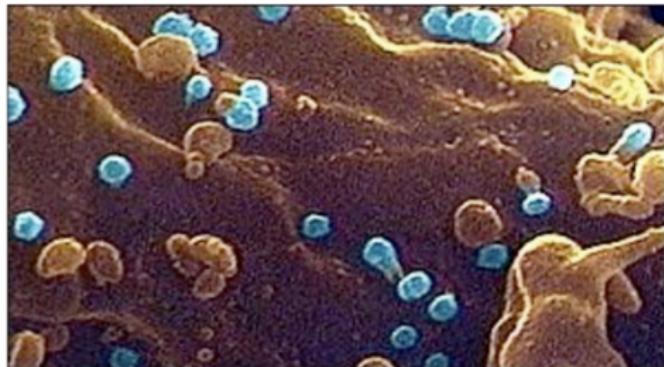
# A calibrated phylogenetic inference

Origin of HIV Epidemic in the Americas, Gilbert et al (2007)



A phylogenetic reconstruction of samples of HIV-1 virus. Each degree one node represents a single infected individual from whom a blood sample has been taken.

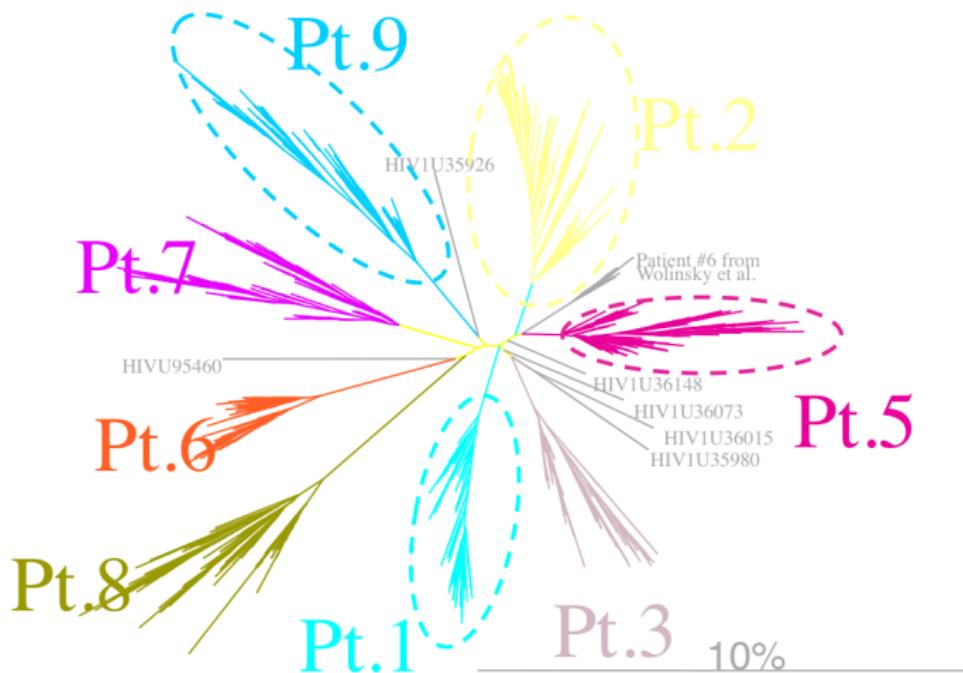
## Human immunodeficiency virus type I(HIV-I)



A single HIV-I infected person has at least  $10^7$  –  $10^8$  infected cells, with each infected cell producing  $\sim 10^3$  viral particles during its life time.

# A tree of HIV sequences from 9 infected patients

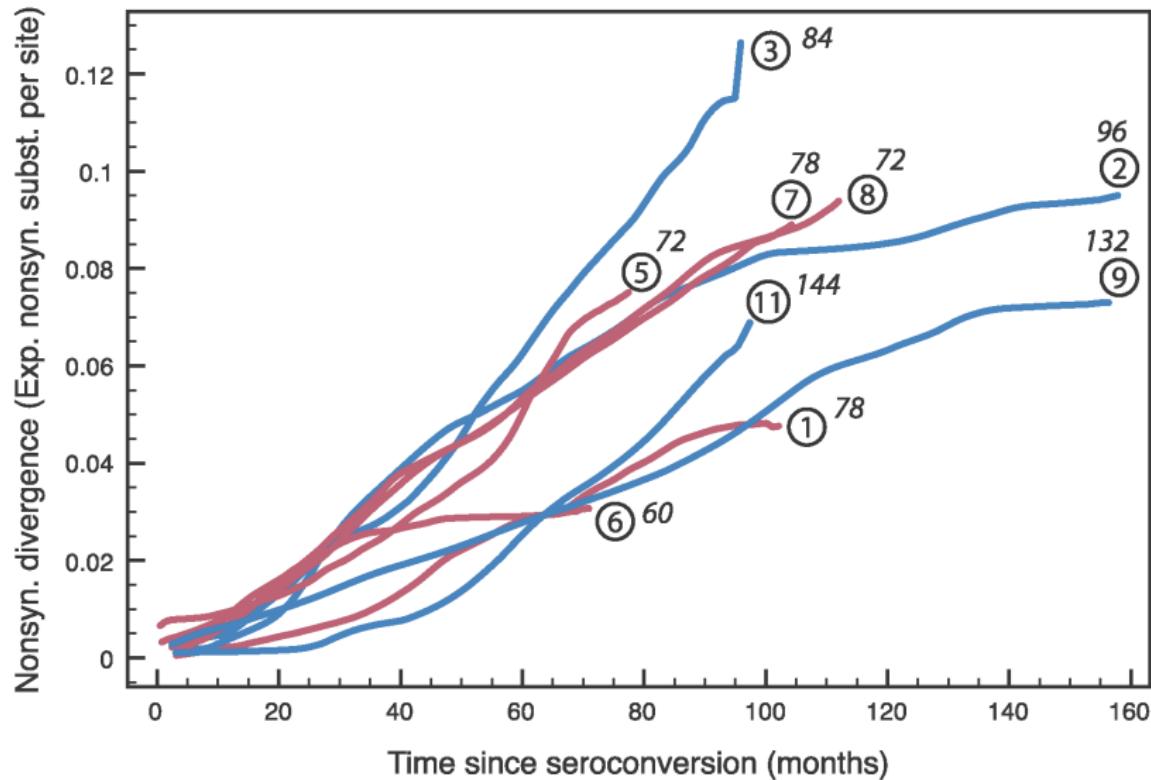
Shankarappa et al (1999)



A phylogenetic reconstruction of samples of HIV-1 virus. Each degree one node represents a single virus particle isolated from a blood sample of one of 9 patients.

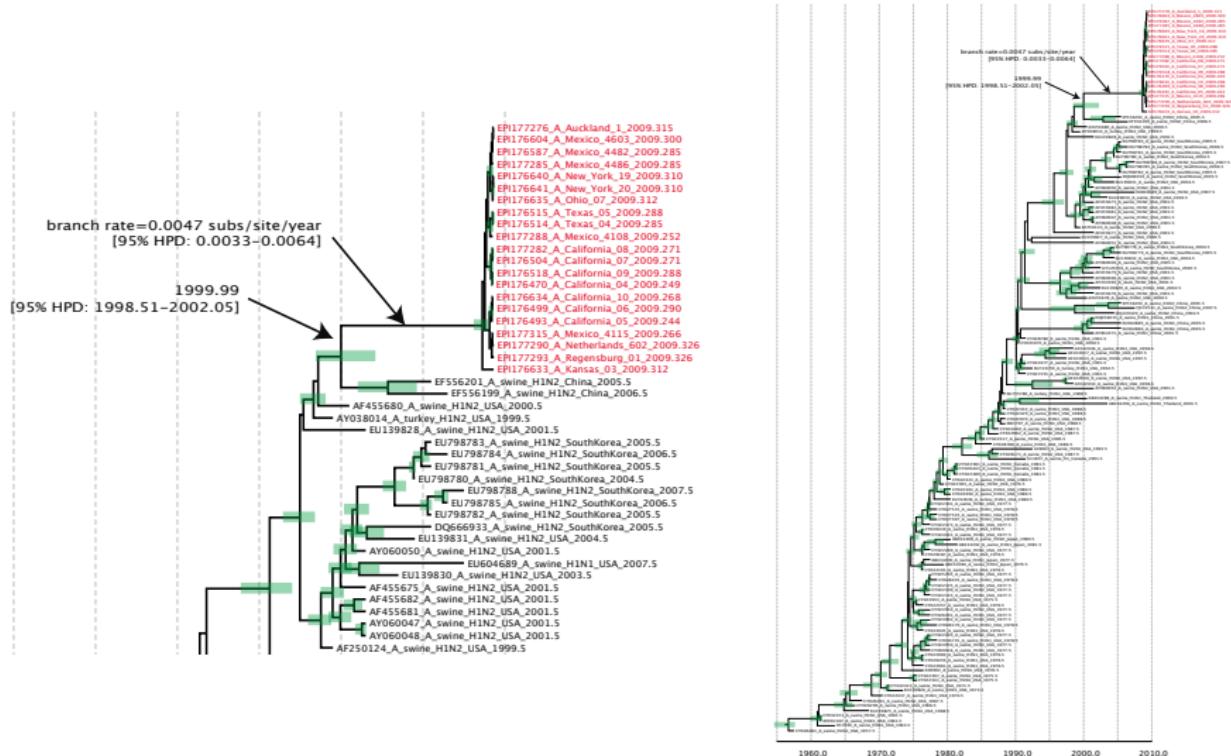
## Estimated accumulation of evolutionary change

Lemey et al (2008)



# On the Origin of 2009 HINI Swine Flu outbreak

<http://tree.bio.ed.ac.uk/groups/influenza/>

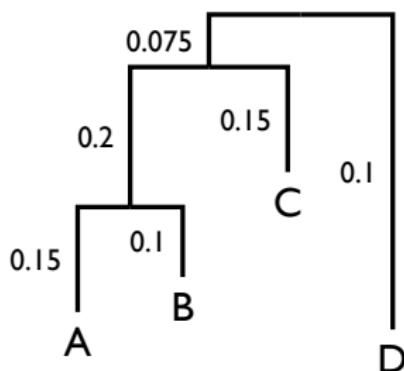


# Relaxed phylogenetics

# Genetic distance = rate × time

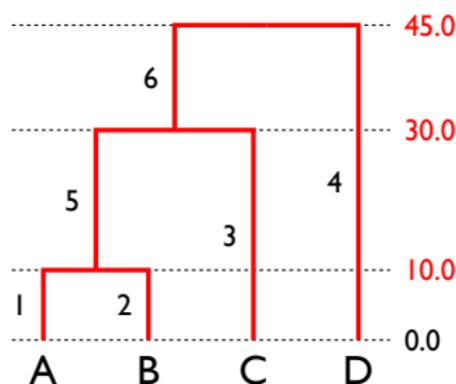
Relaxed molecular clock

$$T = \vec{\mu} * g$$



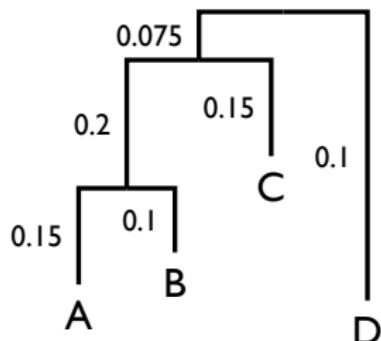
“substitution tree”

evolutionary rates  
substitutions / site / unit time

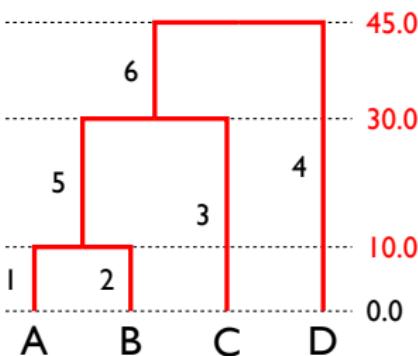


time tree

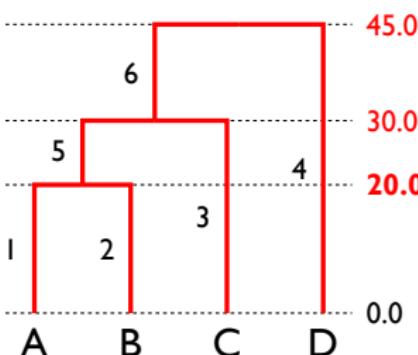
# Nonidentifiability in the relaxed clock



$$= \begin{pmatrix} 0.015 \\ 0.01 \\ 0.005 \\ 0.01 \\ 0.01 \\ 0.005 \end{pmatrix} *$$



$$= \begin{pmatrix} 0.0075 \\ 0.005 \\ 0.005 \\ 0.01 \\ 0.02 \\ 0.005 \end{pmatrix} *$$

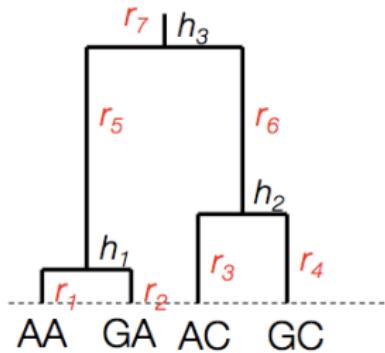


“substitution tree”

evolutionary rates  
substitutions / site / unit time

time tree

## Relaxing the molecular clock



In the field of divergence time estimation auto-correlated relaxed clocks have been considered.

e.g. Thorne et al, 1998:

$$r_i \sim \text{LogNormal}(r_{A(i)}, \sigma^2 \Delta t_i)$$

AC

$$r \sim \text{Exp}(\lambda)$$

We introduce a relaxed clock model in which there is no prior correlation between child and parent rates

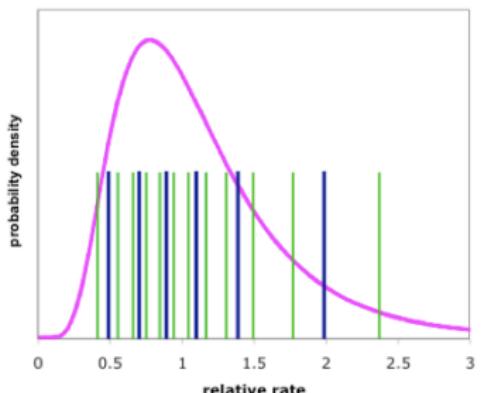
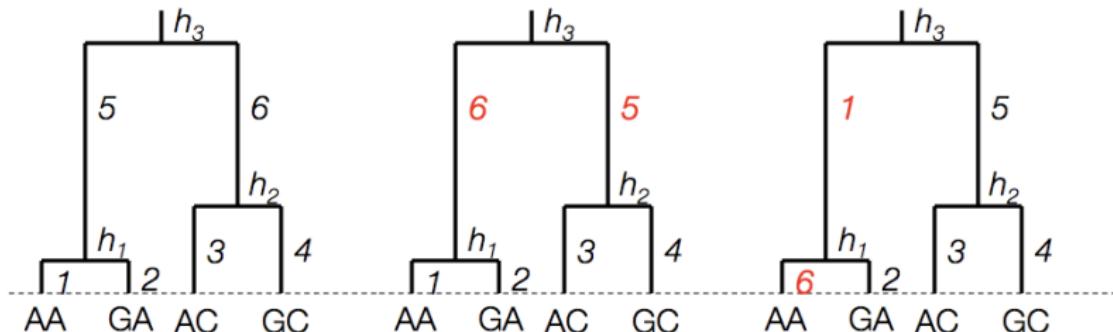
$$r \sim \text{LogNormal}(\mu, \sigma^2)$$

$$r \sim \text{Gamma}(\alpha, \beta)$$

“Un-correlated” or “memory-less” relaxed clocks

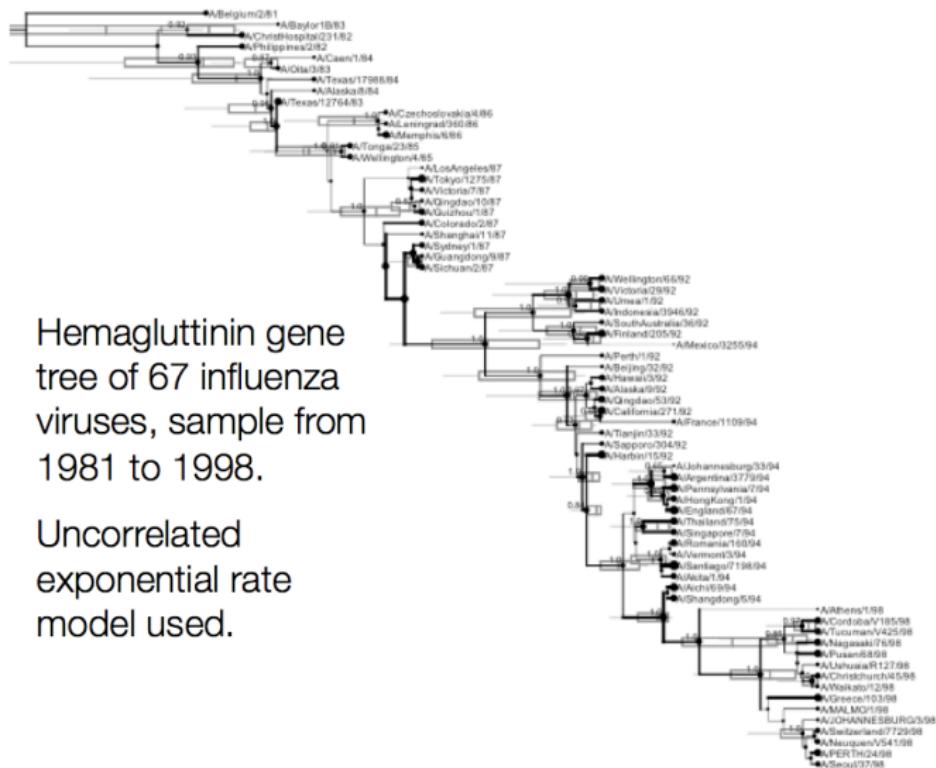
ML

## Sampling branch rates using MCMC



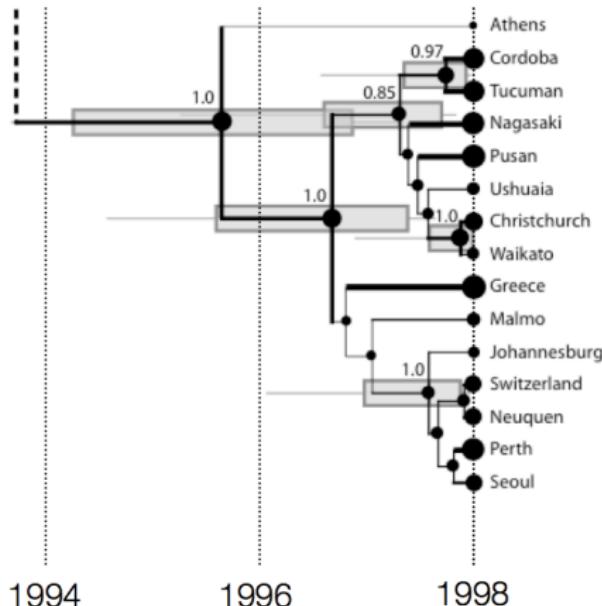
1. Rates are summarized into  $2n-2$  rate categories (e.g. blue is 6 categories; green is 12 categories).
2. Rates categories are sampled during MCMC by two operators:
  1. Random walk operator
  2. Swap operator
3. For purposes of topology changes, rate categories are associated with child node.

## Influenza A gene tree estimated by relaxed molecular clock



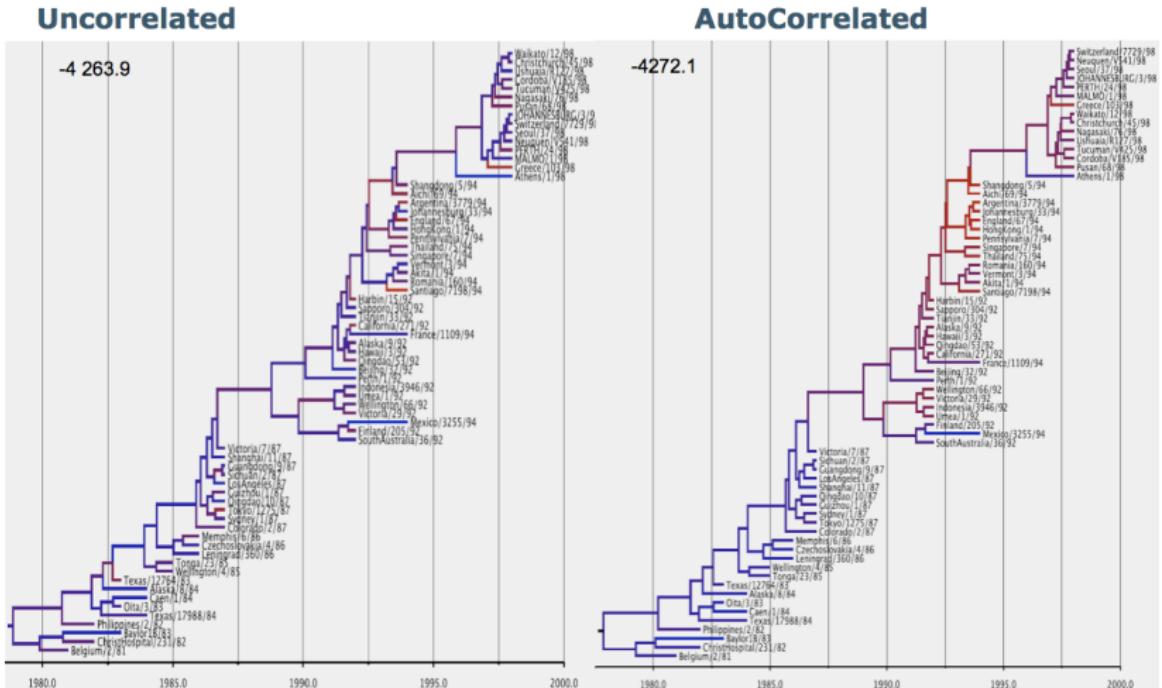
1. Hemagglutinin gene tree of 67 influenza viruses, sample from 1981 to 1998.
  2. Uncorrelated exponential rate model used.

# Influenza A gene tree estimated by relaxed molecular clock

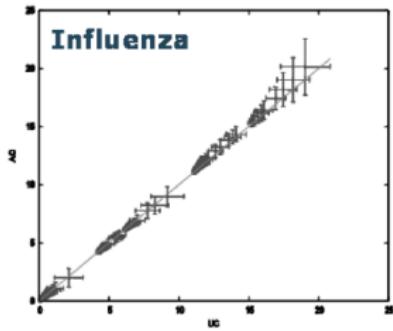
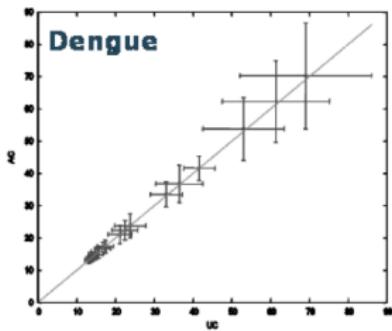
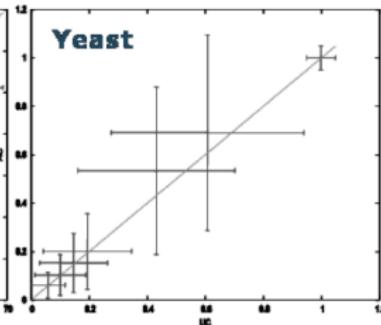
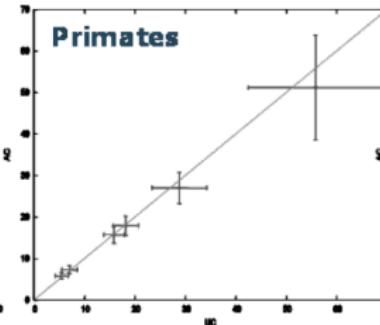
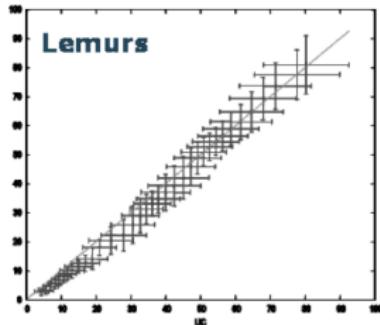


- Box-and-whisker plots show uncertainty in divergence times (only for splits with posterior probability  $> 0.5$ )
- Node size and branch thickness proportional to evolutionary rate.

## Influenza trees under different relaxed clock models



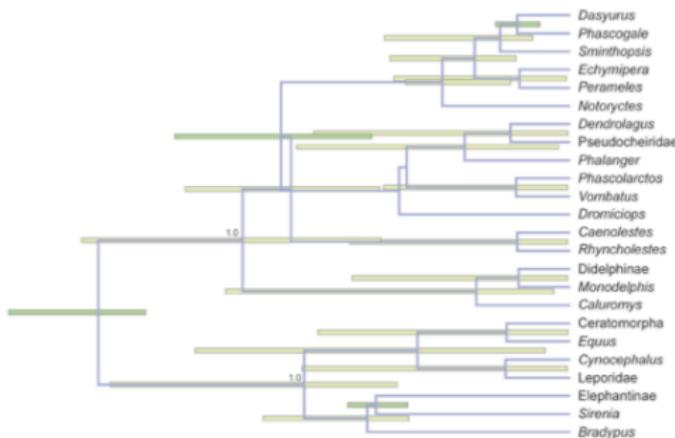
## UC versus AC on five data sets



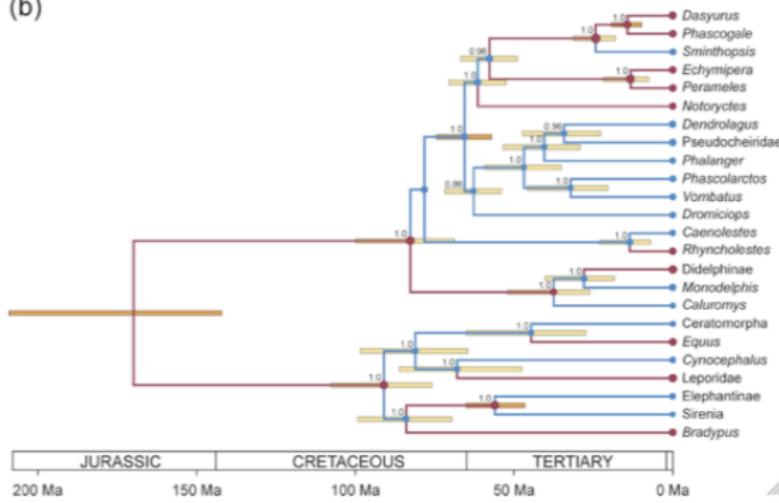
# Prior versus Posterior

Marsupials example  
(24 taxa, 5658 nucleotides)

(a)

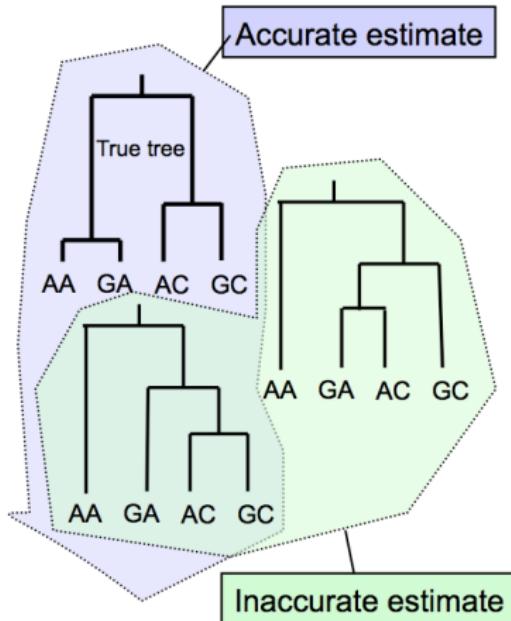


(b)



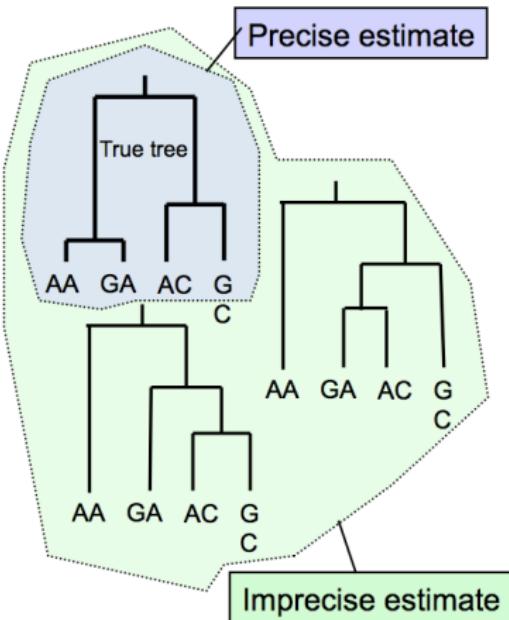
# Accuracy in Bayesian Phylogenetics

- Phylogenetics is an estimation problem, in which the phylogenetic tree topology is the object we wish to estimate.
- The error associated with this estimation can be described by the 95% credible set of trees: the smallest set of trees including 95% of the posterior probability.
- A standard measure of accuracy is the false positive rate. How often do we exclude the true tree from the 95% credible set?



# Precision in Bayesian Phylogenetics

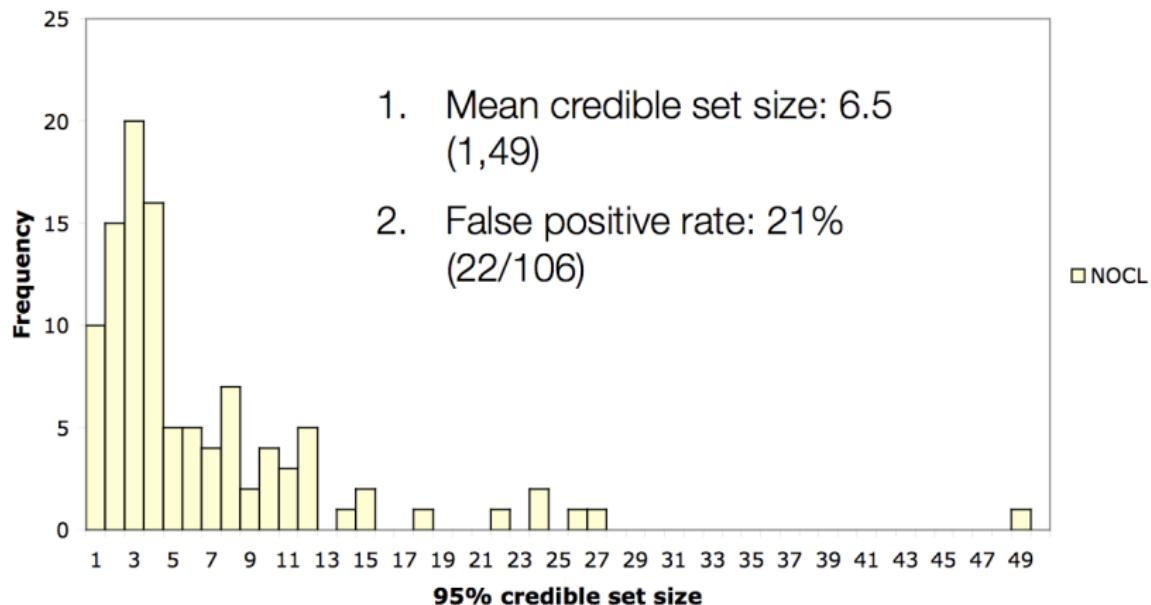
- The precision of an estimate can be described by how much is excluded.
- How small is the 95% credible set of trees?



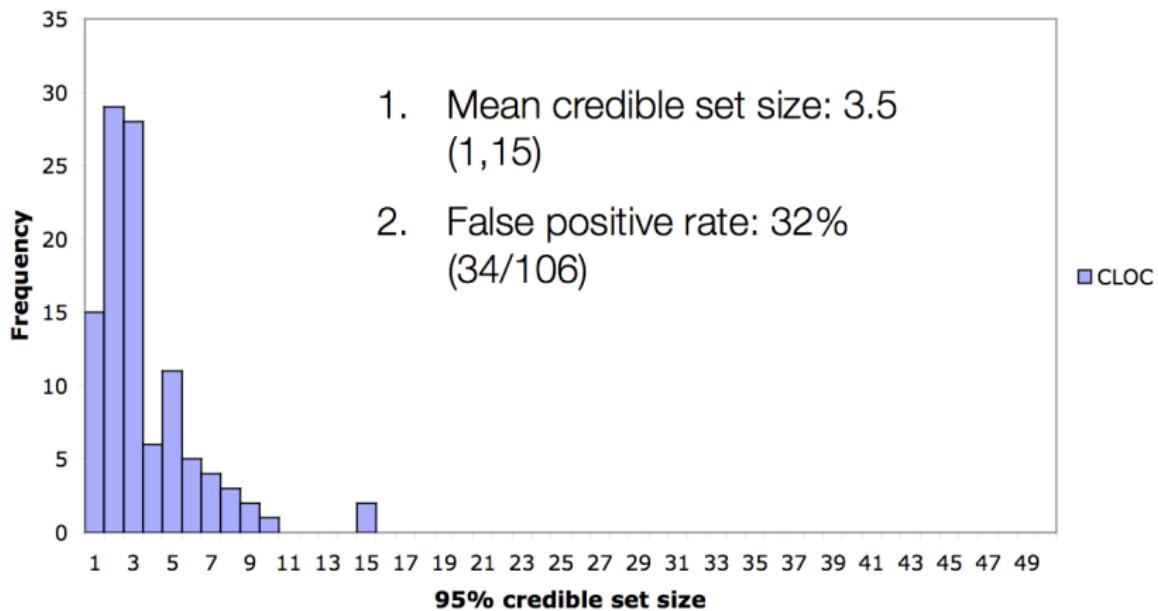
## Testing Accuracy and Precision with **real data**

- Used 106 genes from 8 species of yeast (Rokas *et al*, 2003) and 4 other “phylogenomic” data sets
- For each gene used both MrBayes and BEAST to estimate phylogeny and 95% credible set
- Assumed true tree is the tree estimated using all the concatenated data set.
- Tabulated number of trees in credible set and whether the true tree was in credible set for MrBayes (unconstrained) and BEAST (MLLN and CLOC models)

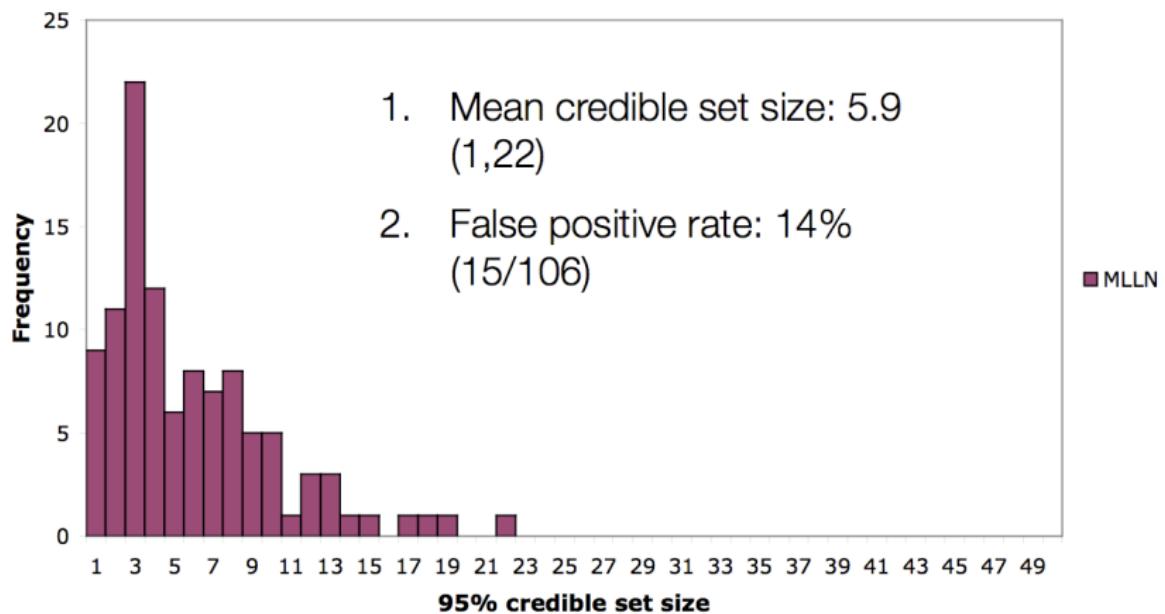
## Rokas data: MrBayes tree estimates



## Rokas data: Strict clock tree estimates from BEAST



## Rokas data: Relaxed clock tree estimates from BEAST



## Summary of Bayesian Accuracy on five large data sets

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Dataset	Sample Size	Average Length	Clock Rejected by LRT	Accuracy (%) (True Tree in 95% Credible Set) <sup>a</sup>
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				CLOC	UCLN	UF
Bacteria	102	170 aa	26%	46.1	<b>48.0</b>	42.2
Yeast	106	1,198 bp	76%	67.0	<b>84.9</b>	79.2
Plants	61	647 bp	67%	<b>91.8</b>	88.5	83.6
Animals	99	197 aa	59%	64.6	<b>69.7</b>	57.6
Primates	500	632 bp	13%	88.8	<b>89.0</b>	88.8

## Summary of Bayesian Precision on five large data sets

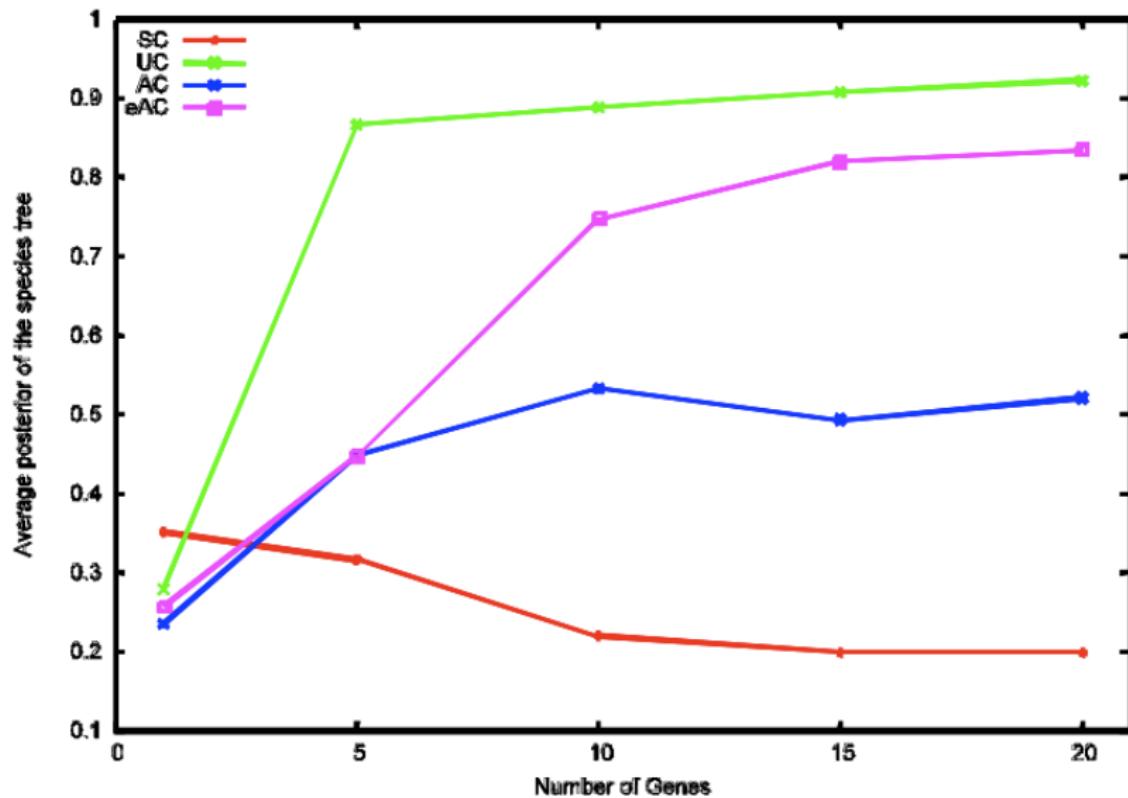
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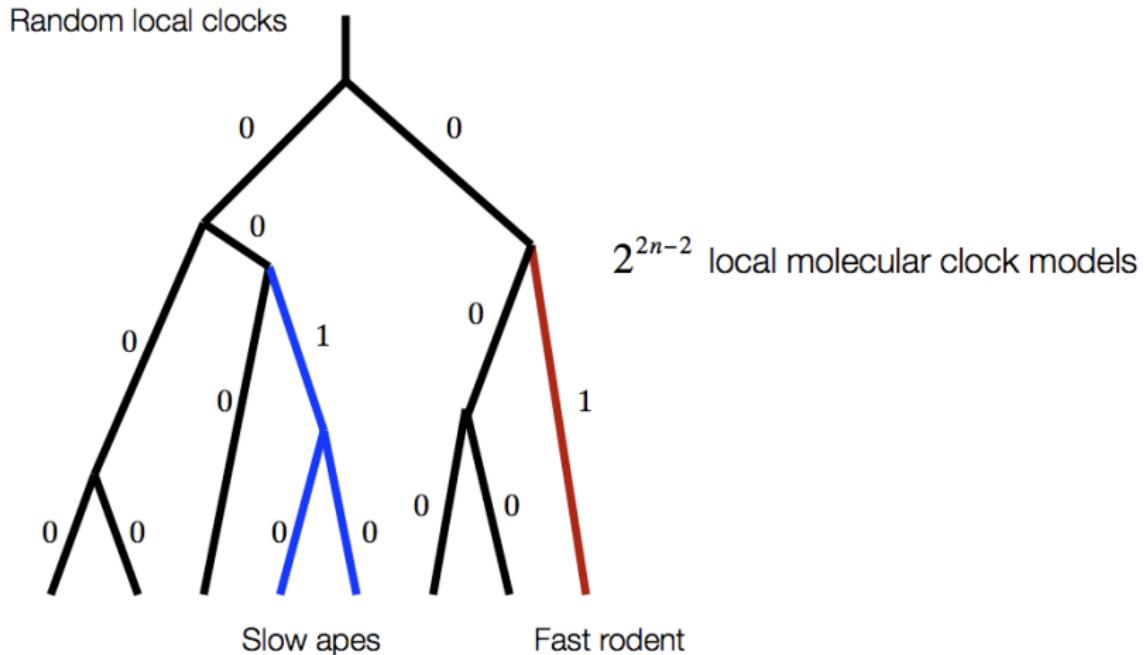
Dataset	Sample Size	Average Length	Clock Rejected by LRT	Precision (Number of Trees in 95% Credible Set) <sup>b</sup>		
				CLOC	UCLN	UF
Bacteria	102	170 aa	26%	5.7	10.3	11.3
Yeast	106	1,198 bp	76%	3.5	5.9	6.5
Plants	61	647 bp	67%	7.5	15.4	9.2
Animals	99	197 aa	59%	5.7	10.2	14.2
Primates	500	632 bp	13%	3.1	3.4	5.1

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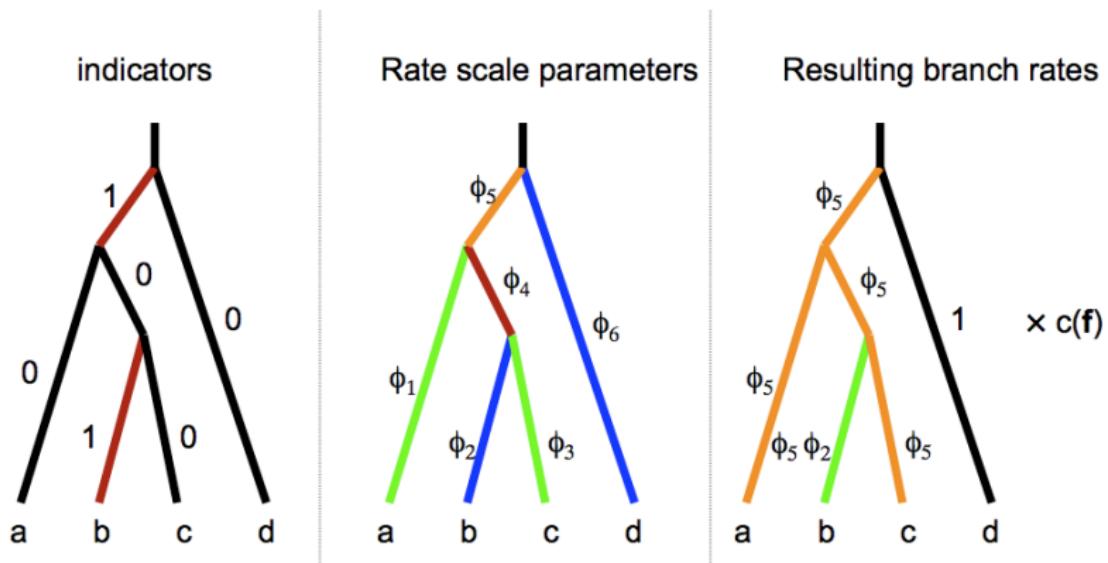
## Increasing the length of the sequence



# Random local molecular clocks

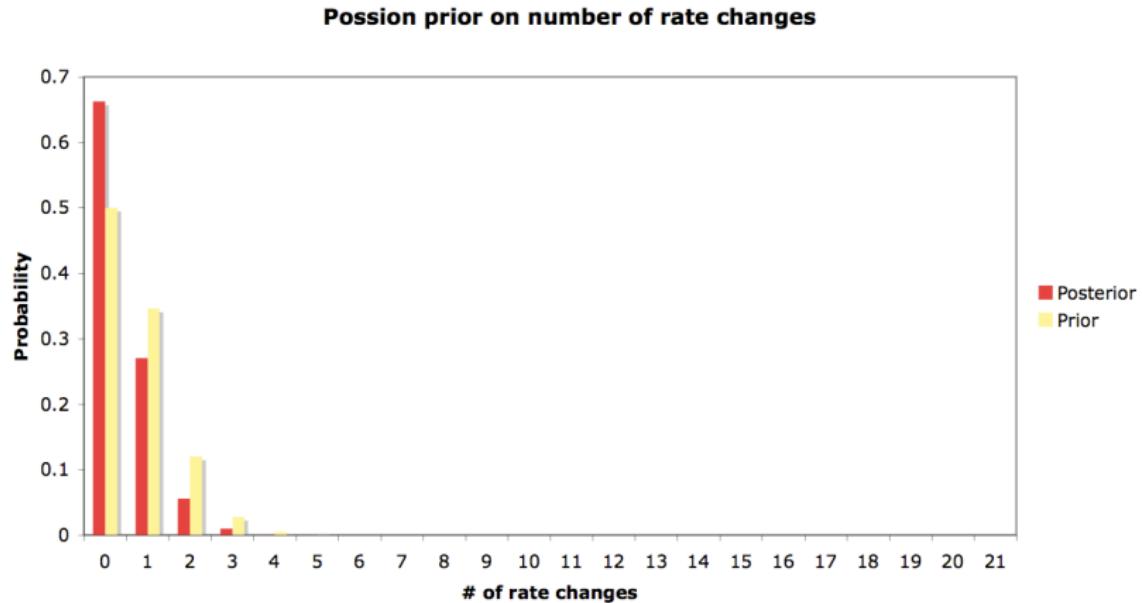


# Random local molecular clocks

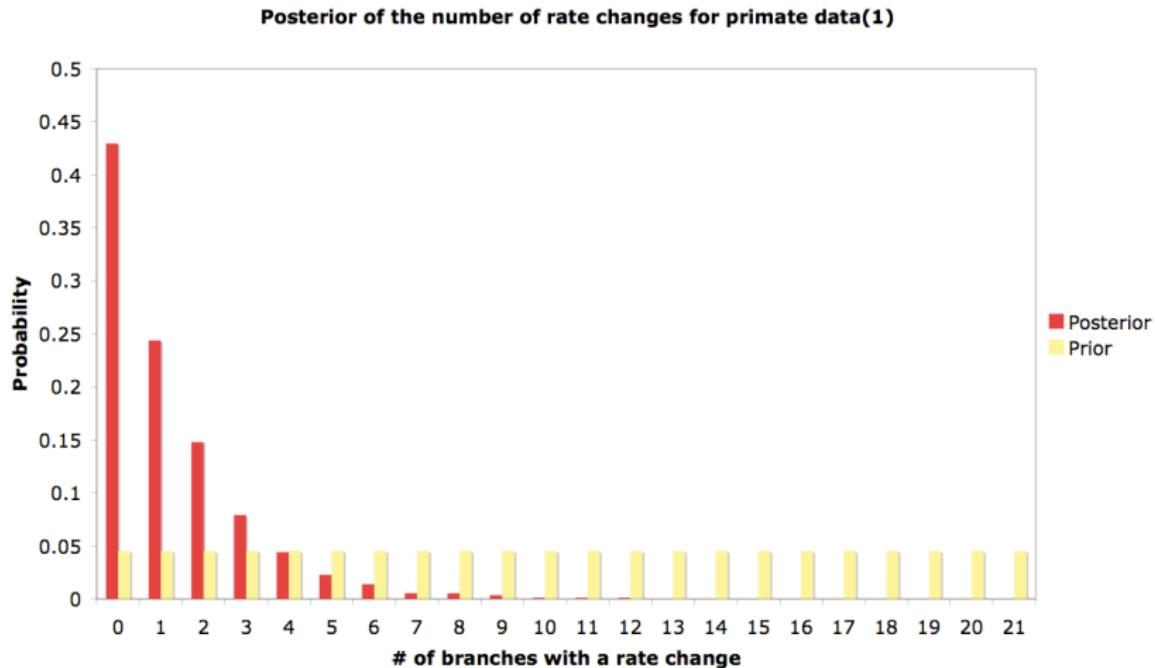


Red/Orange fast, Green/Blue slow

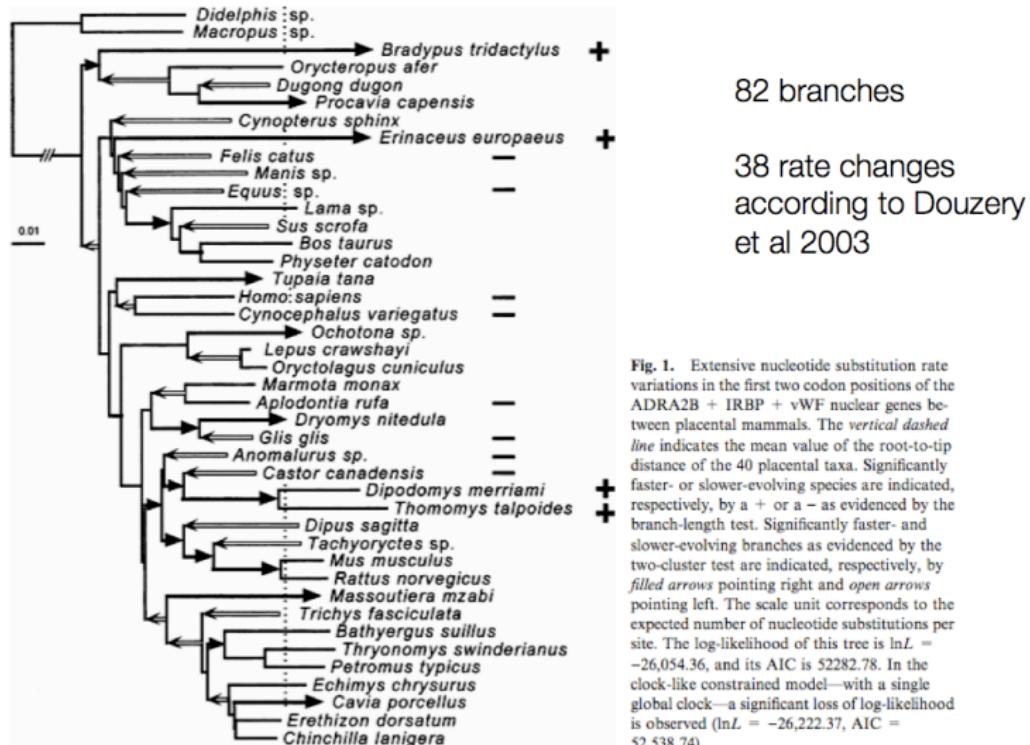
# Primate data set (Poisson prior on # rate changes)



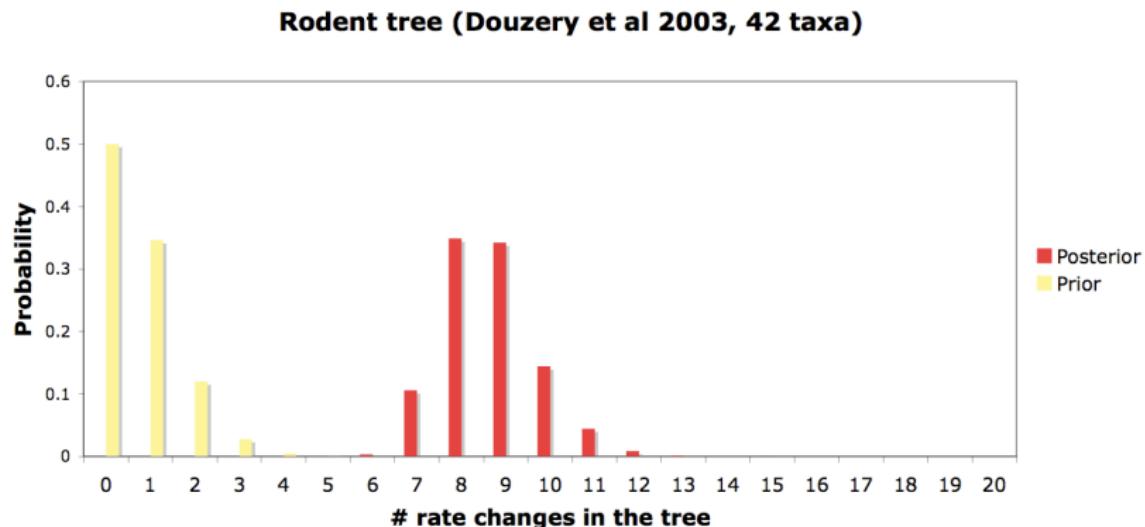
# Primate data set (Uniform prior on # rate changes)



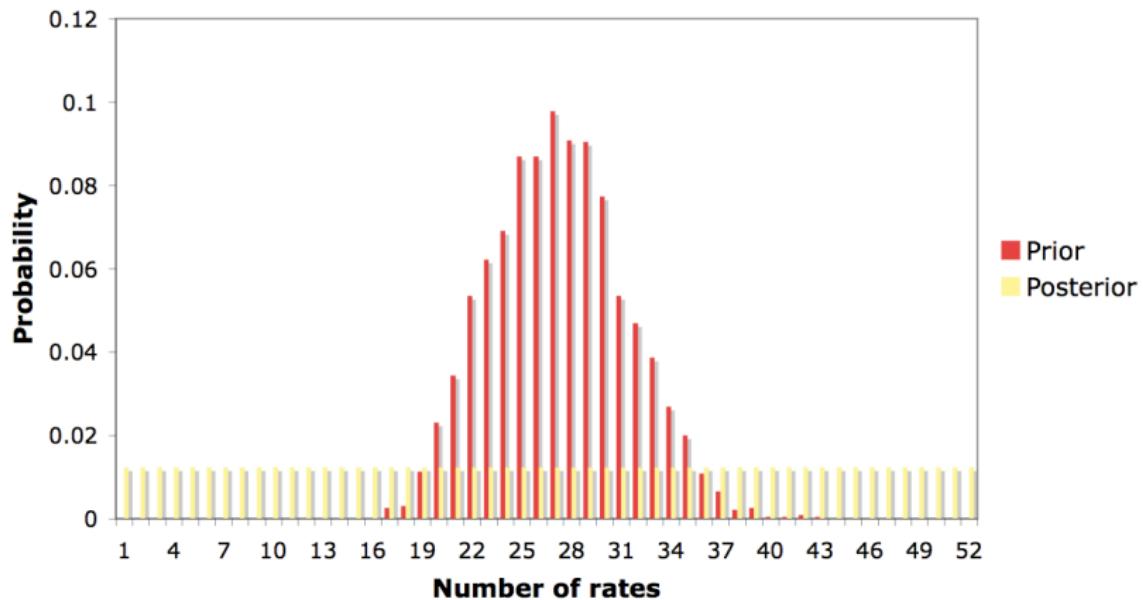
# Rodents (1+2 codon positions from 3 nuclear genes)



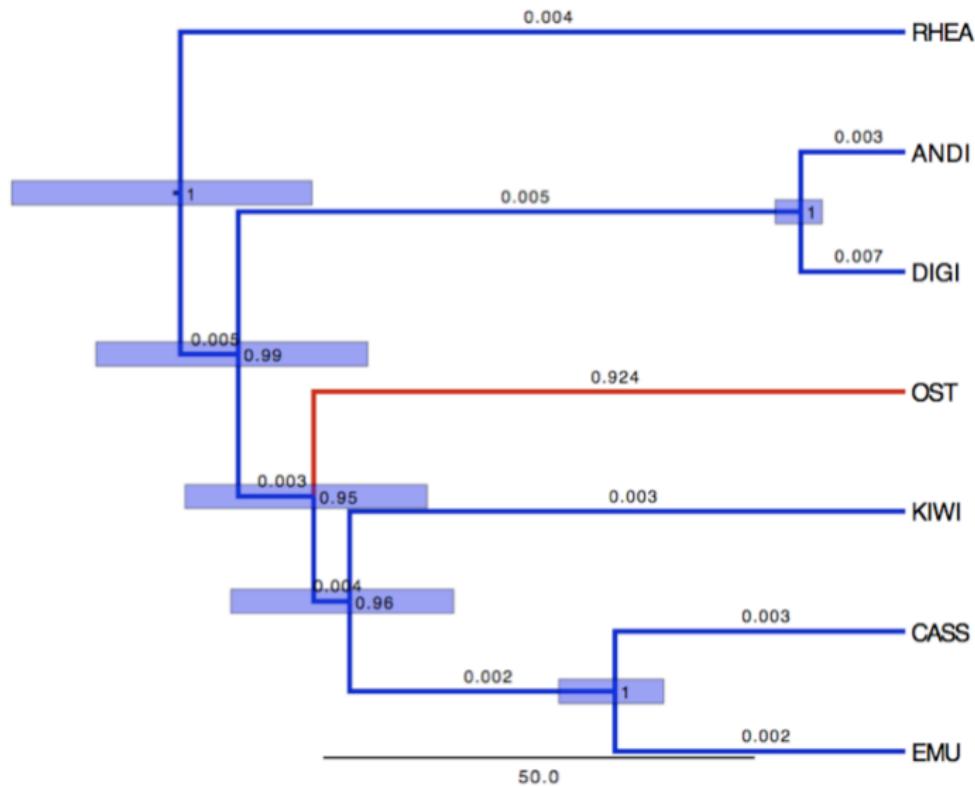
# Rodent data set (Poisson prior on # rate changes)



## Rodents data set (Uniform prior on # rate changes)



# Ratite relaxed clock on full mitochondrial sequences



# Fossilized birth-death trees

Slides by Alexandra Garyushkina  
[sasha.gavyushkina@auckland.ac.nz](mailto:sasha.gavyushkina@auckland.ac.nz)

## Total evidence dating with fossilized birth-death tree prior

- We have molecular data of extant species, morphological data of extant and fossil species and geological ages of fossils (or geological age intervals).
- We want to utilise all this data to learn about evolutionary history of organisms, divergence dates and macroevolutionary parameters.
- Our preferred method is Bayesian phylogenetic inference.

## The calibration method

- An important problem that still requires attention is the divergence dating.
- A common practice in estimating divergence times is using calibration methods when times of internal nodes in the tree are calibrated using fossil records.
- This method has a few drawbacks:
  - usually only the oldest fossil in the clade is used,
  - ad hoc calibration densities,
  - using calibration densities modifies the tree prior,
  - due to computational problems just a few calibration nodes can be used (Heled and Drummond 2012).

## Alternative methods

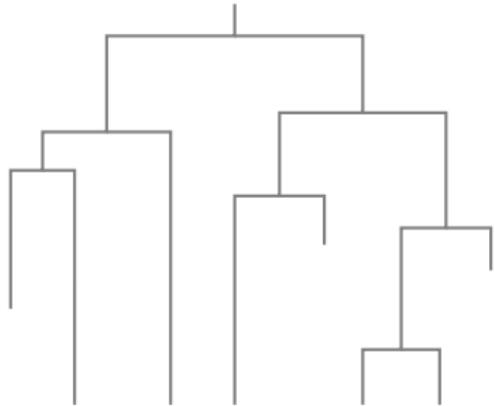
Two main features of the new methods:

- explicit modelling of fossilization events as a part of the tree branching process (Pyron 2011, Heath *et al* 2014),
- utilising all existing data (**total evidence**) in a joint inference (Pyron 2011, Ronquist *et al* 2012).

## Total evidence

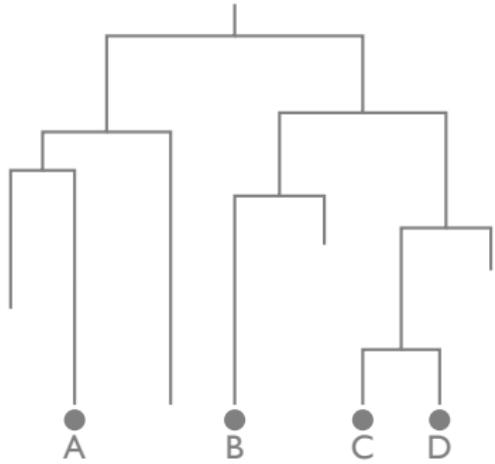
- Pyron 2011 uses Lewis MK model to model evolution of morphological characters.
- Ronquist *et al* 2012, Wood *et al* 2012.
- They used Yule or Uniform tree prior.

## Birth-death-sampling models



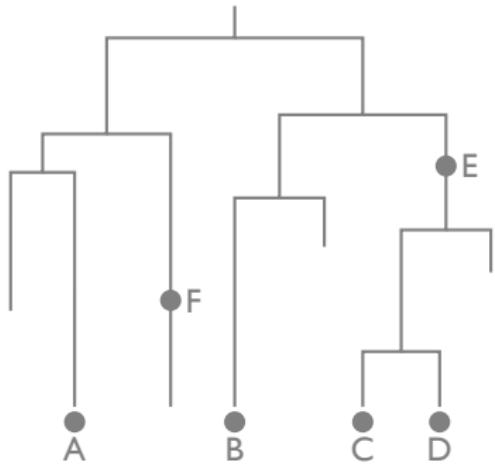
Full tree

## Birth-death-sampling models



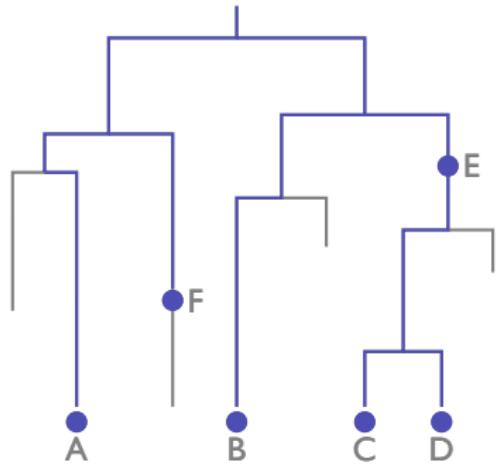
## Full tree

## Birth-death-sampling models



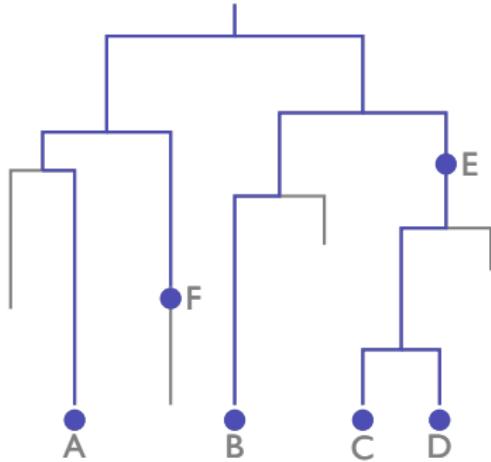
## Full tree

# Birth-death-sampling models

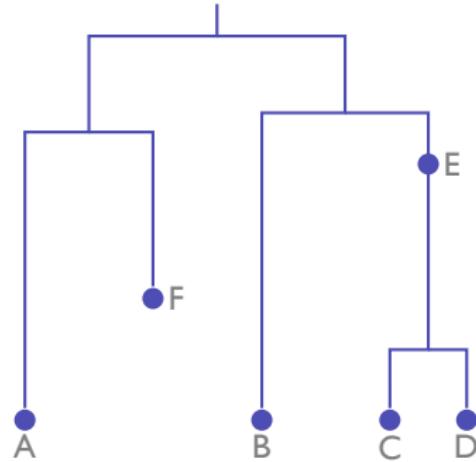


Full tree

# Birth-death-sampling models



Full tree



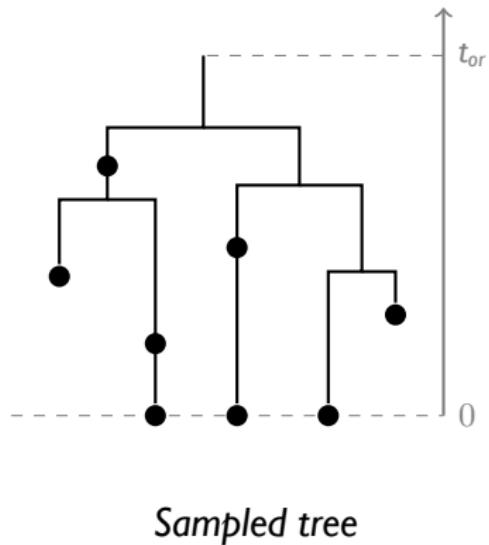
Sampled tree

# Fossilized birth-death model (FBD)

Stadler 2010, Heath et al 2014.

The process starts at time  $t_{or} > 0$  and ends at time zero (present time).

- birth rate  $\lambda$
- death rate  $\mu$
- sampling rate  $\psi$
- sampling at present probability  $\rho$



Model parameters:  $\eta = (t_{or}, \lambda, \mu, \psi, \rho)$ .

All the parameters are identifiable.

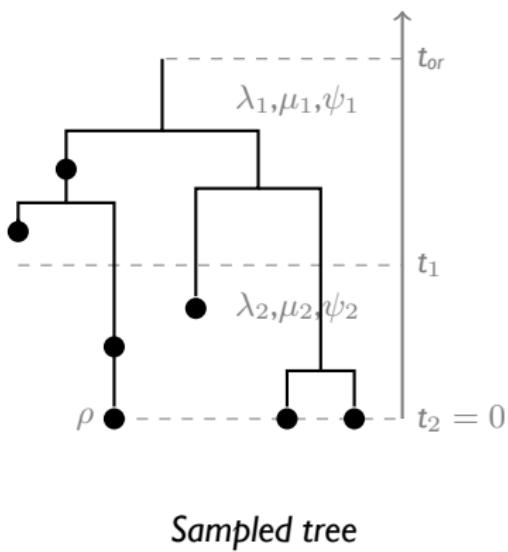
## Fossilized birth-death skyline model (FBD skyline)

Stadler and Kühnert et al 2012, Gavryushkina et al 2014.

There are  $k$  time intervals and parameters remain constants within the intervals but may vary from one interval to another

- birth rates  $\lambda_1, \dots, \lambda_k$
- death rates  $\mu_1, \dots, \mu_k$
- sampling rates  $\psi_1, \dots, \psi_k$
- sampling at time  $t_k$  (present) probability  $\rho$

Model parameters:  
 $\eta = (t_{\text{or}}, \bar{\lambda}, \bar{\mu}, \bar{\psi}, \rho)$



## Heath et al approach and its extensions

Heath et al 2014 first used FBD model to infer divergence times of bears in a Bayesian MCMC framework:

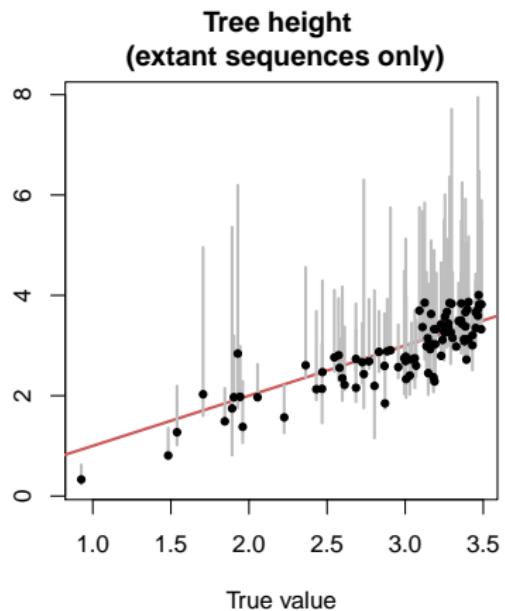
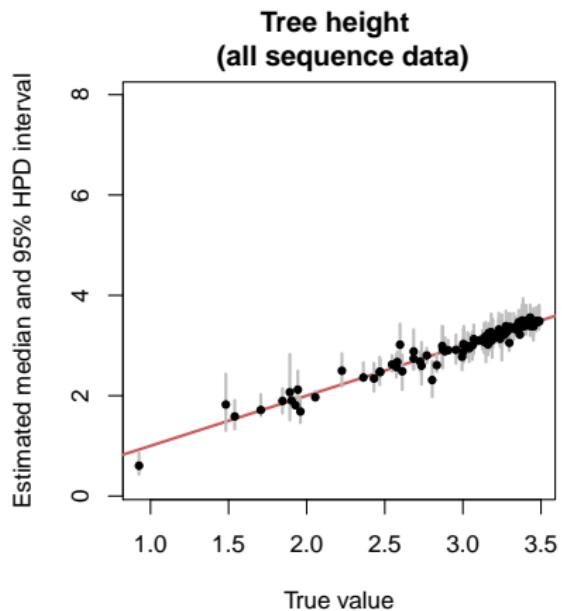
- the topology of extant species is fixed
- they only had occurrence dates of fossil samples
- they fixed  $\rho$  to the truth in the inference

We extend this model in two ways:

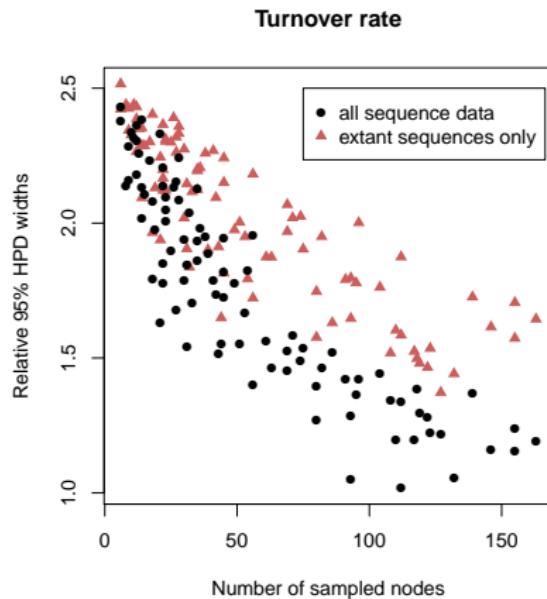
- sampling sampled ancestor trees (Gavryushkina et al 2014)
- incorporating morphological data

SA and Morph-models packages for BEAST2 ([www.beast2.org](http://www.beast2.org))

# Comparative data of fossil tips vs only occurrence dates

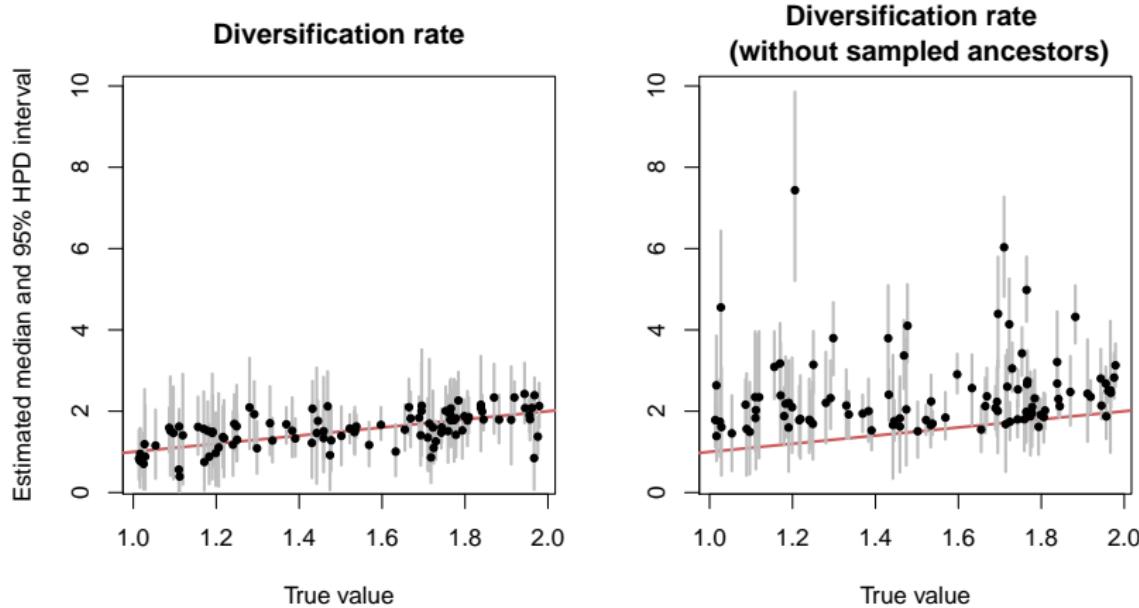


# Comparative data of fossil tips vs only occurrence dates



$$\text{turnover rate} = \frac{\mu}{\lambda}$$

# Biased estimates when not modelling sampled ancestors



$$\text{diversification rate} = \lambda - \mu$$

# Analysis of penguin morphological data

Penguin dataset (Ksepka et al 2012) consisting of morphological data of:

- all extant penguins (19 species)
- 37 fossil species assigned to stratigraphic intervals

Models:

- Lewis MK vs MKv (Lewis 2001)
- Partitions vs single alignment
- Rate variation across sites and across partitions
- Tree prior: FBD and Skyline FBD
- Different tree model parameterisations

# Parameterisations

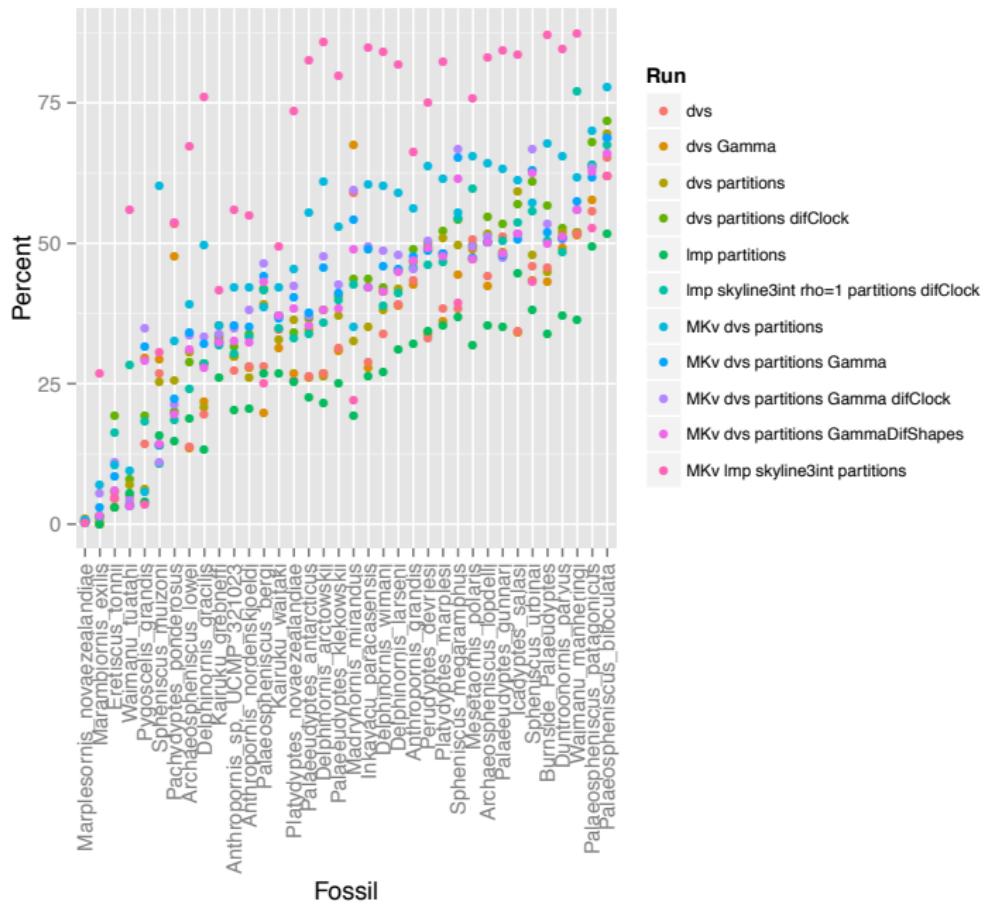
Imp:

birth rate	$\lambda$
death rate	$\mu$
sampling rate	$\psi$
other parameters	$t_{or}, \rho$

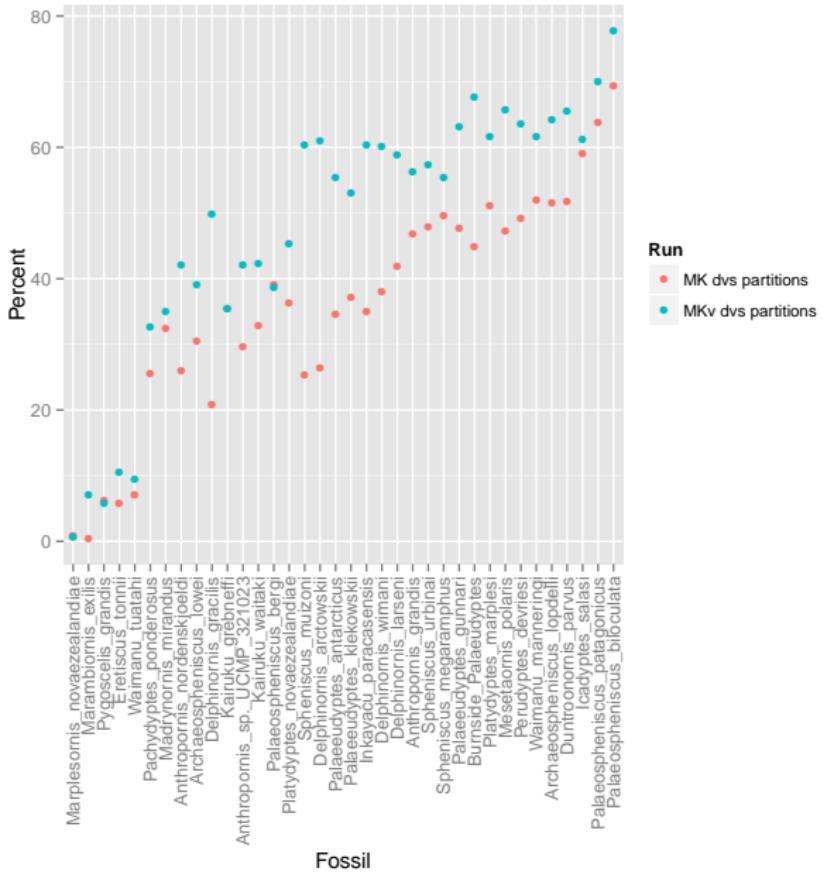
dvs:

net diversification rate	$d = \lambda - \mu$
turnover rate	$\nu = \frac{\mu}{\lambda}$
sampling proportion	$s = \frac{\psi}{\mu + \psi}$
other parameters	$t_{or}, \rho$

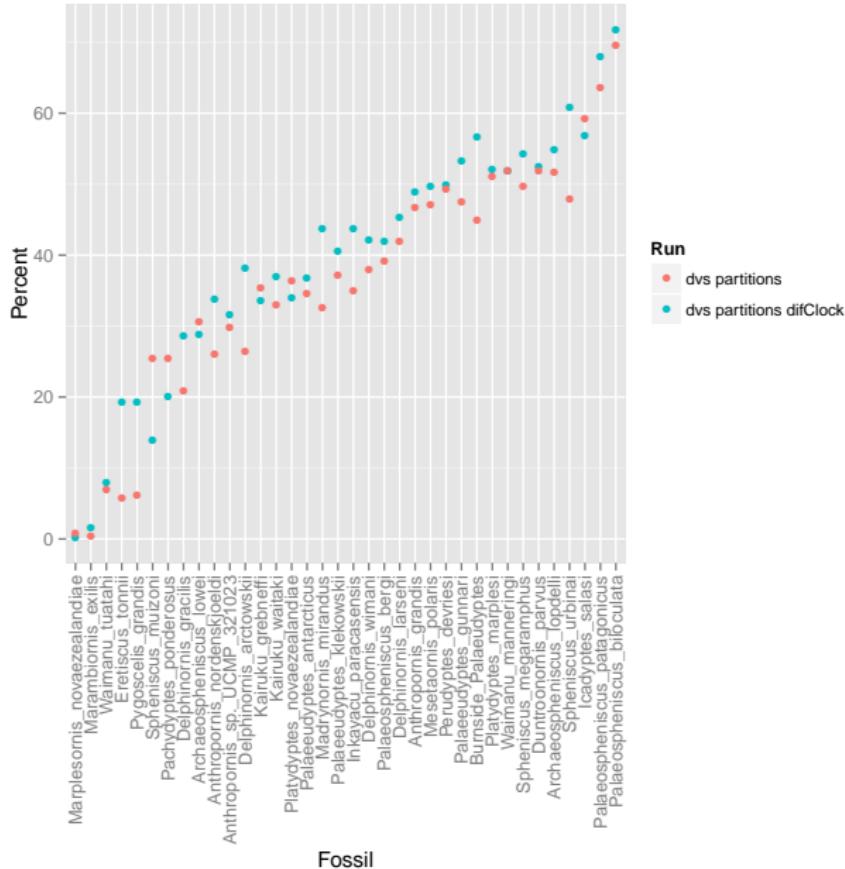
# Posterior probabilities that fossils are sampled ancestors



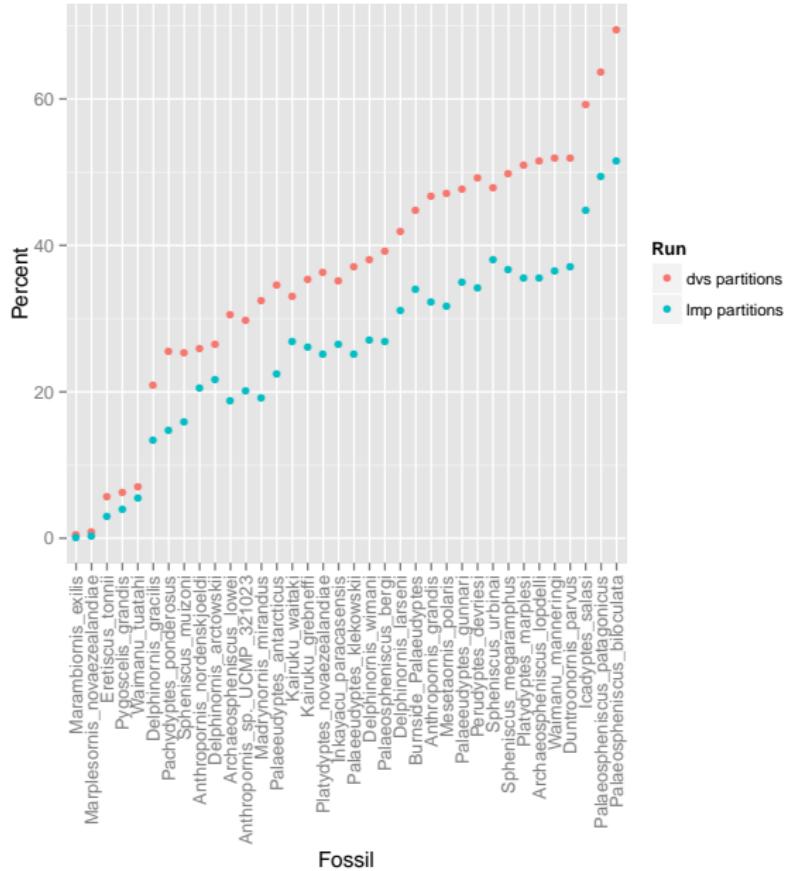
# MK vs MKv



## One clock rate vs different clock rates



# Imp vs dvs



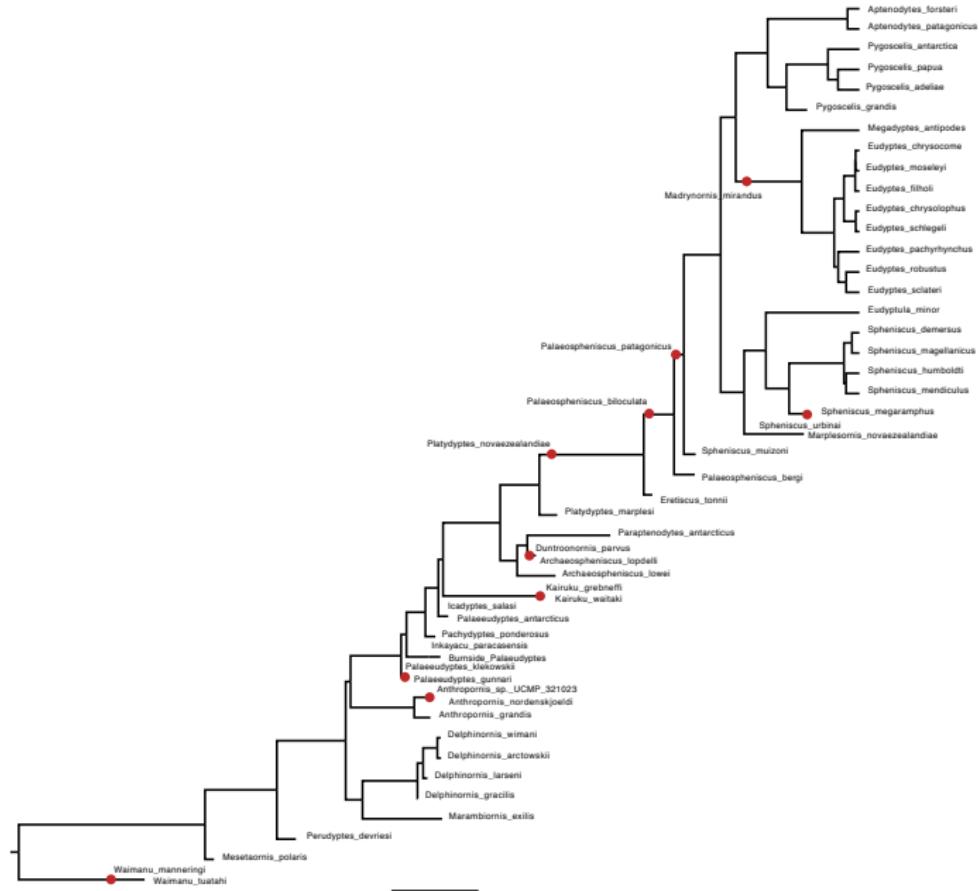
## One analysis of penguin morphological data

Model settings:

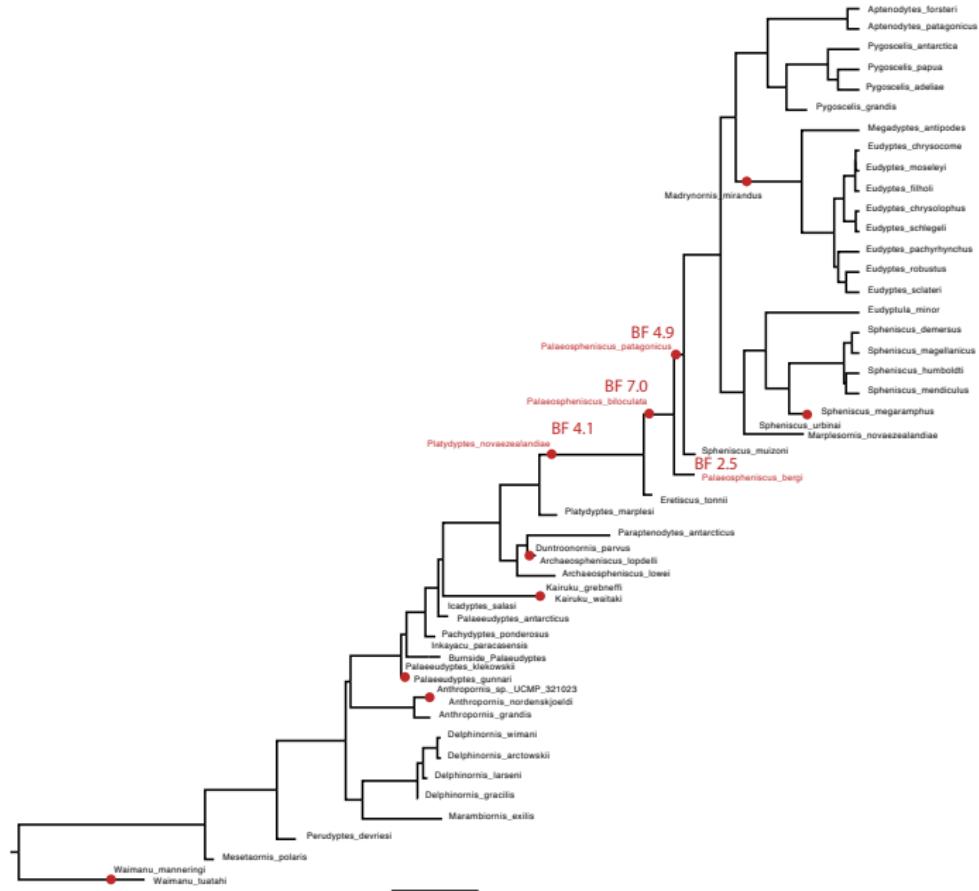
- MK model with partitions,
- Substitution rate variable across partitions,
- FBD Skyline model with 3 equidistant intervals,
- $\rho$  fixed to one.

Summary tree: maximum sampled ancestor clade credibility tree with target heights.

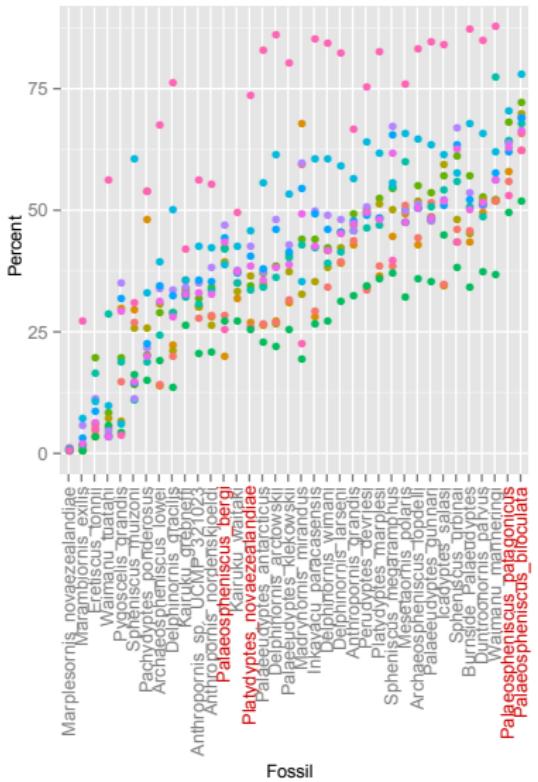
# MCC tree from penguin morphological data analysis



# MCC tree from penguin morphological data analysis



# Posterior probabilities that fossils are sampled ancestors



## Completing the total evidence analysis of penguins

Remains:

- add molecular data to the analysis,
- use a relaxed clock model to estimate divergence times.

## Conclusions

- Relaxed molecular clocks have many benefits over unconstrained models for phylogenetic inference
  - They appear to estimate the phylogenetic tree more accurately on real data sets
  - They automatically provide estimates of a root position, without the need for an outgroup
  - They automatically provide estimates of relative divergence dates, or absolute divergence dates when calibration information is available
- Calibration is hard and interesting
  - Specifying natural means of calibrating phylogenies is subtle
  - Recent methods for including fossil evidence include new tree priors, and opportunities for total evidence dating.
- The geometry of (time) is understudied and its study could lead to new methods for doing phylogenetic inference and posterior post-processing and summary.